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MASTER DEGREE PROJECT

Study the relationship between real exchange rate and interest rate differential – United States and Sweden

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Abstract

This paper uses co-integration method and error-correction model to re-examine the relationship between real exchange rate and expected interest rate differentials, including cumulated current account balance, over floating exchange rate periods. As indicated by the dynamic model, I find that there is a long run relationship among the variables using Johansen co-integration method. Final conclusion is that the empirical evidence is provided to show that our error-correction model leads to a good real exchange rate forecast.

Keywords: exchange rate, interest rate differentials, unit root, co-integration, error correction model and forecast.

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1. Introduction

The study of relationships between real exchange rate and real interest rate has been a interesting topic in the filed of macroeconomics. A number of authors have posited that there is a strong relationship between real exchange rates and real interest rates despite the instability of nominal exchange rates[1]. Two recent work done by Coughlin and Koedijk (1990) and Blundell-Wignall and Browne (1991) found that a long run relationship between the real exchange rate and real interest rate may existed. Ability of Blundell-Wignall and Browne to find co-integration is due to the inclusion of the cumulated current account. Coughlin and Koedijk used monthly data from June 1973 to June 1987 with DF method. The finding of co-integration by them is only for Mark/dollar exchange rate. Blundell-Wignall and Browne used ADF method to test co-integration between the variables over periods from 1963 to 1990 monthly and focused on financial liberalization. They found that financial markets that are almost fully integrated have important implications for real interest rate differentials, real exchange rate behavior and external adjustment. Another new paper, done by Jyh-Lin Wu (1999), implemented Johansen's co-integration method to investigate the relationship between the real exchange rate and expected real interest rate differentials, including a cumulated current account, over the periods from 1974 Q1 to 1994 Q4. He also stated that there is an existence of long-run relationship between the real exchange rate and expected interest rate differentials in a model with current account.

However, many economic professionals did not find that the two variables are co-integrated. Meese and Rogoff (1988) and Edison and Pauls (1993) stated that they did not find a long run relationship between the two variables with using more sophisticated empirical techniques. The model that they investigated is the empirical one that has been proposed by Hooper and Morton. Meese and Rogoff (1988) used Engle and Granger method to test co-integration between the real interest rate

1 See the 1984 Economic Report of the President(3)

differential and real exchange rate over periods from 1974 to 1986. They found that there is a little evidence of relationship between real interest rate differentials and exchange rates and that real disturbance may be a major source of exchange rate volatility. Edison and Pauls (1993) also use the same method to test co-integration between the two variables over periods from 1974 to 1990 and stated that the null hypothesis of non-con-integration can not be rejected with Engle and Granger method. In other words, they also did not find a long-term relationship between them.

In my paper, I applied the Johansen trace method and S&L method to re-examine the long run relationship between these variables. Following Edison and Pauls (1993), the cumulated current account is included in the model.

Purpose of study

The objective of the paper is to re-examine the relationship between the real exchange rate s and real interest rate differentials with using co-integration method and error correction model.

Limitation

In order to study the data over the periods of floating rate, I apply the data starting from 1993 because Sweden government started floating exchange rate since December 1992. Due to a limited and shorter period data from 1993 to 2006, the result obtained could not be pursuable.

Methodology

In the paper, ADF and KPSS methods are used to test for unit roots, and Johansen trace method and S&L are implemented for co-integration test.

Thesis outline

Theoretical view for model is presented in section 1. In this part, I will follow Edison and Pauls (1993) in deriving the relationship between real exchange rates, real interest rate differentials, and cumulated current accounts. In section 2, I presented the data and methodologies. The methodologies include ADF and KPSS for unit root test, and Johansen and S&L for co-integration test. In section 3, I discuss the econometric results. In last part, I come to conclusion.

2. Macroeconomics theory

What determine foreign exchange rate? And could the change in foreign exchange rate be related with real interest rate? These fundamental questions have been hot topics in the world. Now let us review some macroeconomics theory about foreign exchange rate and interest rate.

2.1 Purchasing Power Parity (PPP)

Concept of purchasing power parity has been important explanation for nominal and real exchange rate in the world during 1970s and 1980s. In its absolute version, it states that value of nominal foreign exchange rate equals to the ratio of price level of two countries. It is defined:

$$S = P / P^* \quad (1)$$

Where, S is nominal spot exchange rate and P and P* are domestic and foreign price levels respectively. In its absolute value, PPP states that the change in foreign exchange rate is determined by the relative change in prices in two countries. Value of nominal foreign rate dies not really indicate anything about the "true value" of the currency, or anything related to PPP.[2] A real foreign exchange rate has to be defined:

$$Q = SP / P^* \quad (2)$$

According to Laurence S. Copeland (2000), the general conclusions of PPP can be summarized as follows:

a. In the short run, deviations from PPP are so frequent as to be more or less the norm. They are also very substantial and almost certainly too great to be explained away by international differences in the methods used to collect statistics.
b. The evidence on long-run PPP is mixed. Until recently, the balance appeared to be against, but with the help of powerful new econometric methods, researchers in

[2] See David K.Eiteman Multinational business finance, p107

the last few years have by and large been supportive.

c. In terms of volatility it is unquestionably true that exchange rates have varied far more than prices.

Economists have debated whether PPP applies in the short run, long run or nether. By the end of the 1970s, PPP, at least in the short run was rejected convincingly by the data. Whether PPP in the long run can be rejected is less clear.[3]

2.2 Interest Rate Parity

Interest rate parity provides a link between interest rate and exchange rate. It states: the difference in the national interest rates for securities of similar risk and maturity should be equal to, but opposite in sign to, the forward rate discount or premium for the foreign currency, except for transaction costs.[4] Two versions: covered interest rate parity and uncovered interest rate parity.

2.2.1 Covered Interest Rate Parity (CIP)

Covered interest rate parity states that interest rate difference between two countries equals to percentage difference in spot exchange rate and forward exchange rate. A basic CIP model can be defined as follows:

$$(1+i) = S(1+i^*)/F \qquad (3)$$

Where, S is spot exchange rate, F is forward exchange rate and i is interest rate. Star indicates foreign variable. If the parity condition does not hold, there will be an arbitrage opportunity.

2.2.2 Uncovered Interest Rate Parity (UIP)

Uncovered interest parity is an important building block for macroeconomic analysis

[3] See Coughlin, C.C. and Kiedijk, K.(1990)
[4] See David K.Eiteman Multinational business finance, p114

in open economies. It provides a simple relationship between domestic the interest rate, foreign interest rate, and the expected rate of change in the spot exchange rate between the two countries.[5] A model without arbitrage is defined as follows:

$$(1+i_{t,k}) = \frac{S_t(1+i^*_{t,k})}{E_t\left[S_{t+k}\right]}. \qquad (4)$$

Where, $E_t\left[S_{t+k}\right]$ is expected spot exchange rate at time t +k based on time t. Others remain same as equation (3).

UIP has been a challengeable model in macroeconomic field. Although validity of UIP is strongly challenged by the empirical evidence, at least in short run, some papers states that its retention increases with longer time.[6]

2.2 Fisher Effect

The fisher effect states that nominal interest rates in one country are equal to the required real rate if return plus compensation for expected inflation. The model is defined as follows:

$$i = r + \pi + r\pi \qquad (5)$$

Where, i is nominal rate of interest, r is the real rate of interest and π is the expected rate of inflation. An approximation is defined as follows:

$$i = r + \pi .$$

[5] See Peter Isard (1996)
[6] See Peter Isard (1996), Menzie and Meredith (2002)

3. Theoretical view for model

I will follow Edison and Pauls(1993) in deriving the relationship between real exchange rates, real interest rate differentials, and cumulated current accounts. Uncovered interest rate is defined as following：

$$(1+i_{t,k}) = \frac{S_t(1+i^*_{t,k})}{E_t[S_{t+k}]} \tag{1}$$

Take log form for both side of the equation, the uncovered interest parity with a risk premium is defined as follows:

$$s_t = E_t(s_{t+k}) + i_{t,k} - i^*_{t,k} - \rho_t \tag{2}$$

s = log of spot exchange rate（foreign currency per dollar）

E(x) = expected value of any future variable x based on information at time t,

$i_{t,k}$, $i^*_{t,k}$ = nominal interest rate on bond issued at time t and matures at time t+k, star denotes foreign variable,

ρ_t = risk premium, which is assumed to be covariance stationary.[7]

The real exchange rate is defined as

$$q_t = s_t + p_t - p_t^* \tag{3}$$

Or

$$E_t(q_{t+k}) = E_t(s_{t+k}) + E_t(p_{t+k}) - E_t(p_{t+k}^*) \tag{4}$$

Where, q is log of the real exchange rate, p is log of domestic price levels, and p* is log of foreign price levels.

Now I combine equation (2) with equation (4) and get the following formula:

$$s_t = E_t(q_{t+k}) + E_t(p_{t+k}) - E_t(p_{t+k}^*) + i_{t,k} - i^*_{t,k} - \rho_t \tag{5}$$

The expected future price at time t is defined as:

$$E_t(P_{t+k}) = P_t * E_t(1+\pi_{t,k}) \tag{6}$$

I take equation (6) as log forms, using approximation, as following:

[7] The assumption is also adopted by Edison and Pauls (1993).

$$E_t(p_{t+k}) = p_t + E_t(\pi_{t,k}), \quad E_t(p^*_{t+k}) = p_t^* + E_t(\pi^*_{t,k}) \qquad (7)$$

Applying the fisher effect equation to obtain an expression for expected real rates of interest as following:

$$E_t(r_{t,k}) = i_{t,k} - E_t(\pi_{t,k}), \quad E_t(r^*_{t,k}) = i^*_{t,k} - E_t(\pi^*_{t,k}) \qquad (8)$$

Where, $E_t(\pi_{t,k})$ is expected inflation rates from time t to time k. And then I combine equation (3), (5), (7) and (8) and obtain:

$$q_t = E_t(r_{t,k}) - E_t(r^*_{t,k}) + E_t(q_{t+k}) - \rho_t. \qquad (9)$$

In order to make a relationship between real exchange rates and expected real interest rate differentials, it is necessary to model the expected long run real exchange rates ($E_t(q_{t+k})$). I will follow Meese and Rogoff (1988) and assume that expected long run real exchange rates equal to the equilibrium real exchange rates. Moreover, Hooper and Merton (1982) indicated that the equilibrium real exchange rates can have a linear function with a constant and cumulated current accounts, i e, $E_t(q_{t+k}) = \alpha + \beta ccad_t$. Where, α is a constant, and ccad equals to difference in the share of the cumulated current accounts relative to GDP. Therefore, equation (9) can become:

$$q_t = \alpha + E_t(r_{t,k}) - E_t(r^*_{t,k}) + \beta(ccad)_t - \rho_t \qquad (10)$$

Or

$$q_t = \alpha + \theta * rd_t + \beta(ccad)_t - \rho_t, \qquad (11)$$

Where, $rd = E_t(r_{t,k}) - E_t(r^*_{t,k})$, assuming that coefficient, θ, is existed in the equation. My task is to study relationship between real exchange rate and expected real interest rate differentials with co-integration test. I will use Johansen and S&L method to test co-integration. VECM will be applied if the co-integration is found between the variables. If not, VAR model will be instead of VECM.

4. Methodology and data

4.1 Data

I will apply the quarterly economic data from 1993 to 2006. The data include: the bilateral SEK/Dollar exchange rates, consumer price indices, long term 10 years bond yields as nominal interest rates, current accounts, and GDP for US and Sweden. The cumulated current account balances are created assuming the cumulated current accounts of US and Sweden were in balances of 1992.Q4; the current accounts were then accumulated as 1993.Q1. The real interest rate is the ex-post interest rate, in which the expected inflation is proxied by the actual inflation by assuming a perfect foresight of the agent. In my study, all the data are obtained from Ecowin.

4.2 Unit root test

Most economic time series are non-stationary. Whether a variable is stationary is important when we make analysis for time series. If we use non-stationary variables to make a regression model, there might be a spurious regression. A spurious regression has a high R^2 and t-statistics that appear to be significant, but the results are without any economic meaning. Existence of a unit root indicates a variable is non-stationary, and therefore the variable has to be integrated of order one, denoted I (1) in order to be a stationary variable. If taking first difference does not produce a stationary variable, the variable will be integrated of order two. Two different unit root tests will be used which are ADF and KPSS tests.

4.2.1 The Augmented Dickey – Fuller test (ADF)

The ADF test is the extended version of the Dickey – Fuller test because the regression has been augmented with the lagged changes. The ADF statistic, used in the test, is a negative number. The more negative it is, the more there is no existence of a unit root. If a unit root is found, the time series are not stationary. We have to take difference for the time series until they become stationary. The ADF models as following as[8]:

A model with constant and time trend:

$$\Delta \mathbf{y}_t = \alpha + \beta t + \theta y_{t-1} + \sum_{i=1}^{k} \gamma \Delta y_{t-i} + \varepsilon_t$$

A model with constant:

$$\Delta \mathbf{y}_t = \alpha + \theta y_{t-1} + \sum_{i=1}^{k} \gamma \Delta y_{t-i} + \varepsilon_t$$

A restricted model:

$$\Delta \mathbf{y}_t = \theta y_{t-1} + \sum_{i=1}^{k} \gamma \Delta y_{t-i} + \varepsilon_t$$

Where the null hypothesis is H_0: $\theta = 0$, and the alternative is $\mathrm{H}_1 : \theta < 0$. If null hypothesis is accepted, the time series are not stationary because there is existence of a unit root. If alternative is accepted, the time series are stationary.

4.2.2 KPSS test

Another possibility for testing unit root is KPSS. The test is calculated by RATS4[9]. If it is assumed that there is a linear time trend, Data generation process is assumed as following:

$y_t = \beta t + x_t + \varepsilon_t$, where, $x_t = x_{t-1} + \eta_t$, $\varepsilon_t \sim$ iid $(0, \sigma^2{}_\varepsilon)$ and $\eta_t \sim$iid $(0, \sigma^2{}_\eta)$. Null hypothesis is H_0 : $\eta_t{}^2 = 0$ and alternative hypothesis is H_1 : $\eta_t{}^2 > 0$. If H_0 is accepted, y_t is stationary. If H_1 is accepted, there is existence of a unit root, and we have to make difference for the time series. Test is given in Kwiatkowski et al (1992)

[8] See Boo, Sjoo 2004, P4
[9] See Boo, Sjoo my book, p110.

$$KPSS = \frac{1}{T^2}\sum_{t=1}^{T} S_t{}^2 \hat{\sigma}^2{}_\infty \,,$$

Where $S_t = \sum_{j=1}^{t} \hat{\omega}_j$ with $\hat{\omega}_t = y_t - \bar{y}$ and $\hat{\sigma}^2{}_\infty$ is an estimator $\sigma^2{}_\infty$. If y_t is stationary, S_t is I (1) and the number in the numerator if the KPSS is an estimator of its variance.[10]

4.3 Con-integration

Co-integration describes a long run linear combination of many series. Variables are co-integrated when a linear combination among them is stationary even though the variables are not stationary. However, a regression on non-stationary series will produce spurious correlation among the variables. If single variables in a model have different trend processes, they can not stay in a fixed long run relation to each other, implying that one is not able model the long, and there is no valid base for inference based on standard distributions[11].. Therefore it is necessary to use stationary variables when we make regression among the variables. If co-integration is found among the variables, Vector Error Correction Model (VECM) will be applied instead of Vector Autoregressive Regression (VAR) Model.

Two steps to test co-integration:

1. Determine the degree of integration in every variable by using unit root test.
2. Estimate the co-integration regression and test integration.

In my co-integration test, I follow the model in JMulTi software:

$$y_t = D_t + x_t$$

Where, y_t is K-dimensional vector of variables, D_t is a deterministic term, and x_t is a VAR (p) process with VECM representation:

$$\Delta x_t = \Pi x_{t-1} + \sum_{j=1}^{p-1} \Gamma \Delta y_{t-j} + \mu_t$$

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10 See Lutkepohl and Kratzig applied time series ecnomics 2004 p65
11 see Engle; Granger, 1987, P.265

Where, μ_t, is a vector white noise process. Π, is co-integrating rank of variables. Co-integration test checks the following hypotheses:

$H_0(0): rk(\Pi) = 0$ Versus $H_1(0): rk(\Pi) > 0$,

$H_0(1): rk(\Pi) = 1$ Versus $H_1(1): rk(\Pi) > 1$,

$\vdots$

$H_0(K-1): rk(\Pi) = K-1$ Versus $H_1(K): rk(\Pi) = K$

I will use Johansen trace tests and Saikkonen & Lutkepohl to test co-ingration in my thesis.

4.3.1 Johansen trace test

Johansen (1988, 1991, 1992, 1994, and 1995) has proposed likelihood ratio tests which are known as trace test because of the special form of the test statistics. The distributions of the test statistics under their respective null hypotheses is determined by the deterministic terms. Here three basis modeling forms are defined:

1. Restricted mean term and no liner trend

The deterministic term, e.g., $D_t = \mu_t$

And the DGP of the y_t may be written as following:

$$\Delta y_t = \Pi^* \begin{bmatrix} y_{t-1} \\ 1 \end{bmatrix} + \sum_{j=1}^{p-1} \Gamma \Delta y_{t-j} + \mu_t$$

Where, $\Pi^* = [\Pi : \upsilon_0]$ is (K*(K+1)) with $\upsilon_0 = -\Pi\mu_0$. The test statistics is received by reduced rank regression applied to this model with rk $(\Pi^*) = r_0$ [12]

2. Constant and linear trend

Here $D_t = \mu_0 + \mu_1 t$, and the DGP may be written as

$$\Delta y_t = \nu + \Pi^+ \begin{bmatrix} y_{t-1} \\ t-1 \end{bmatrix} + \sum_{j=1}^{p-1} \Gamma \Delta y_{t-j} + \mu_t$$

[12] see Johansen (1995)

Where, $\Pi^{+} = \alpha[\beta' : \eta]$ is (K*(K+1)) matrix of rank r_0 with $\eta = -\beta'\mu_1$. The test is based on the model.[13]

3. Trend orthogonal to co-integration relations

Here, $D_t = \mu_0 + \mu_1 t$ again, and the DGP may be written as

$$\Delta y_t = \nu + \Pi y_{t-1} + \sum_{j=1}^{p-1} \Gamma \Delta y_{t-j} + \mu_t \ .$$

The test is base on the model (see Johansen (1995)). In the case Saikkonen and Lutkepohl (2000a) argue that it is not meaningful to test $H_0(K-1) : rk(\Pi) = K-1$ versus $H_1(K) : rk(\Pi) = K$.

4.3.2 Saikkonen & Lutkepohl Test

Saikkonen & Lutkepohl Test (2000a, b, c) is based on a reduced rank regression of the model:

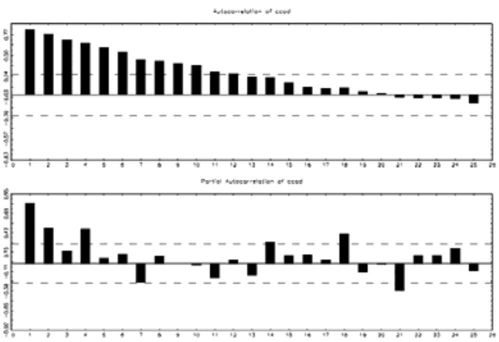
$$\Delta\hat{x}_t = \Pi\hat{x}_{t-1} + \sum \Gamma_j \Delta\hat{x}_{t-j} + \hat{\varepsilon}_t$$

Where $\hat{x}_t = y_t - \hat{D}_t$ and $\hat{D}_t$ is the estimated deterministic term. The deterministic term is estimated by the GLS procedure. In the test possible options (a constant, a constant and a linear trend, a trend orthogonal to co-integration relations) are same as the option in Johansen trace test.

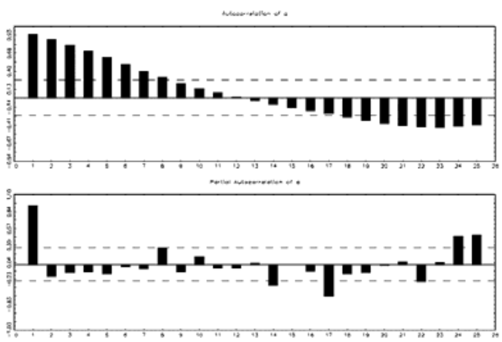
[13] See Johansen (1994, 1995)

5. Empirical investigation

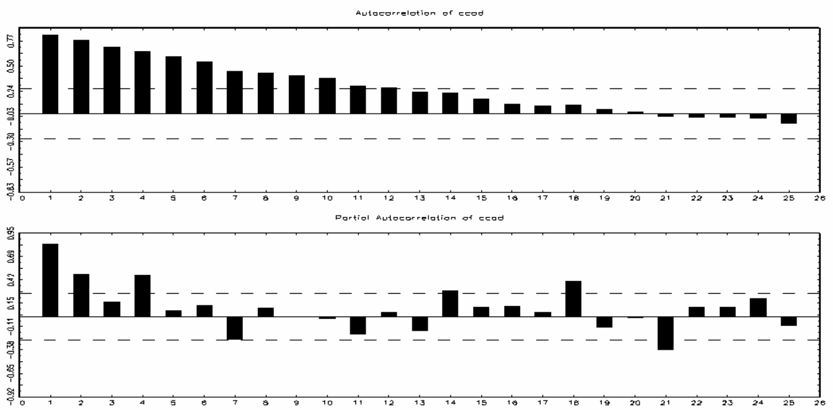
Before implementing ADF and KPSS test, I use the autocorrelation graph to make analysis for different time series. It is known that if the autocorrelation are significant for most of the lags and the values never die out, this indicates the time series are not stationary. As be shown in graph 1, 2 and 3, these 3 time series are not stationary because the values from the graphs never die out. The results will be confirmed in root unit test in following section by ADF and KPSS test. Therefore, differencing the time series are necessary in order to make them stationary.



Graph 1: autocorrelation of ccad



Graph 2: autocorrelation of q



Graph 3: autocorrelation of rd

5.1 Unit roots test

In order to confirm the result obtained by the previous graph, I use ADF and KPSS test to analyze the time series. I choose the statistics value at 5% significance level to compare with that of test statistic. If the value of test statistics is within the range of value at 5% significance, I will accept null hypothesis (existence of a unit root). If not, alternative hypothesis is accepted. When implementing KPSS test, I get the optimal lags by using the formula: $l_q = q(T/100)^{1/4}$ With the formula, lags are 10 when implementing KPSS test.

Table 1

Variables	Test	SC	AIC	Optimal lag	Test statistics	1%	5%	10%
ccad	**ADF**	**3**	**3**	**3**	**-1.8579**	**-3.43**	**-2.86**	**-2.57**
	KPSS			**10**	**0.6288**	**0.739**	**0.463**	**0.347**
ccad_dl	**ADF**	**2**	**2**	**2**	**-8.1417**	**-3.43**	**-2.86**	**-2.57**
	KPSS			**10**	**0.1566**	**0.739**	**0.463**	**0.347**
q	**ADF**	**0**	**1**	**0**	**-1.2358**	**-3.43**	**-2.86**	**-2.57**
	KPSS			**10**	**0.2698**	**0.739**	**0.463**	**0.347**
q_dl	**ADF**	**0**	**0**	**0**	**-5.5574**	**-3.43**	**-2.86**	**-2.57**
	KPSS			**10**	**0.1447**	**0.739**	**0.463**	**0.347**
rd	**ADF**	**1**	**1**	**1**	**-1.0854**	**-3.43**	**-2.86**	**-2.57**
	KPSS			**10**	**0.4116**	**0.739**	**0.463**	**0.347**
rd_dl	**ADF**	**1**	**1**	**1**	**-7.2169**	**-3.43**	**-2.86**	**-2.57**
	KPSS			**10**	**0.1136**	**0.739**	**0.463**	**0.347**

For cumulated current account, value of statistics, with 3 lagged difference (suggestion by Schwarz Criterion), given by ADF is -1.8579. Because the value is more than -2.86 (5% significance level), there is existence of a unit root. With KPSS test, the value of test statistics is more than the value of 5% significance level (0.6288 versus 0.463). The both test confirmed that the time series is not stationary. Therefore first differencing it is necessary. After taking first difference, I found that ADF test value is less than value of 5% significance level (-8.1417 versus -2.86) and that KPSS value is less than value of 5% significance level(0.0959 versus 0.143), it means that the time series become stationary. Normally, real exchange rates are affected by the season. Therefore I choose seasonal dummy when implementing ADF test. We found that the statistics value given by ADF is -1.2358, which is obviously more than -2.86 (5% significance level). Although KPSS test indicated that the time series is stationary because statistics value is less than that of 5% significance level, I still assumed that the real exchange rate is not stationary. In other words, I have to make first difference for real exchange rate, i.e., I (1). After taking first difference, both tests confirmed that the time series are stationary. Finally I test real interest rate differentials. As can be indicated in the table 1, ADF test showed that real interest rate differentials are not stationary (-1.0854 versus -2.84), whereas KPSS test (0.4116 versus 0.463) indicated that it is stationary. Although the two results are different, I still assumed that the time series are not stationary because both the graph and ADF indicated that it is not stationary. Therefore it is necessary to taking first difference for the time series, i.e., I (1). The time series became stationary after it was taken first difference because both ADF and KPSS indicated that it is so. Because the time series have same integration order, I will test co-integration among the variables in the following section.

5.2 Co-integration

In this part, co-integration test is performed for real exchange rates (q) and expected real interest rate differentials (rd) and cumulated current account (accad), real exchange rates (q) and expected real interest rate differentials (rd). I will check the equation:

$$q_t = \alpha + \theta * rd_t + \beta(\text{ccad})_t - \rho_t$$

If co-integration exists among the variables, the equation will be hold. Johansen trace test and Saillonen and Lutkepohl (S&L) test are implemented for co-integration analysis. The results are showed in Table 2 and Table 3, where the numbers of lag are suggested by Schwarz and Akaike Info Criterion.

Table 2: the model with q and rd

Variables	Test	No. of lag	H0	LRs	90%	95%	99%
q,rd	Johansen	1	r=0 r=1	99.07 27.33	17.98 7.60	20.16 9.14	24.69 12.53
		4	r=0 r=1	22.69 9.25	17.98 7.60	20.16 9.14	24.69 12.53
	S&L	1	r=0 r=1	37.10 18.92	10.47 2.98	12.26 4.13	16.10 6.93
		4	r=0 r=1	18.44 5.38	10.47 2.98	12.26 4.13	16.10 6.93

Table 3: the model with q,rd and accd

Variables	Test	No. of lag	H0	LRs	90%	95%	99%
q,rd and accd	Johansen	1	r=0 r=1 r=2	175.19 89.86 28.48	32.25 17.98 7.60	35.07 20.16 9.14	40.78 24.69 12.53
		9	r=0 r=1 r=2	48.19 12.76 3.73	32.25 17.98 7.60	35.07 20.16 9.14	40.78 24.69 12.53
	S&L	1	r=0 r=1 r=2	110.80 43.61 22.96	21.76 10.47 2.98	24.16 12.26 4.13	29.11 16.10 6.93
		9	r=0 r=1 r=2	25.47 6.48 0.19	21.76 10.47 2.98	24.16 12.26 4.13	29.11 16.10 6.93

From the table 2, the null hypothesis can be rejected at any condition with suggestion by Schwarz criterion because the likelihood ratios are more than critical values. Although the results suggested by AIC are not perfect, the null hypothesis also can be rejected. All these facts indicate that there is the existence of co-integration between the two variables. As of model with q, rd and accd, the null hypothesis also can be rejected; it indicates existence of co-integration among the three variables.

5.3 Error correction model

According to Granger's representation theorem, if a co-integrating relationship exists among a set of I (1) series, then a dynamic error correction representation of the data also exists[14]. A real exchange rate equation is defined as following:

$$\Delta q_t = \alpha + \sum_{i=0}^{n} \beta_{1i} \Delta q_{t-i-1} + \sum_{i=0}^{n} \beta_{2i} \Delta rd_{t-i} + \sum_{i=0}^{n} \beta_{3i} \Delta ccad_{t-i} + \lambda_k \phi_{t-1} + \varepsilon_t$$

According to AIC, I choose ten lags when estimate model. Actually, not all coefficients in the above equation are statistically significant. In my study, I restrict the coefficients with 5% significant level and get model as following:

$$\Delta q_t = \underset{(-5.187)}{-0.715}\Delta q_{t-1} \underset{(-3.470)}{-0.510}\Delta q_{t-2} \underset{(-2.418)}{-0.383}\Delta q_{t-3} \underset{(-2.400)}{-0.320}\Delta_{q-7} \underset{(-2.069)}{-0.274}\Delta q_{t-8} \underset{(-3.349)}{-0.384}\Delta q_{t-10}$$
$$\underset{(-2.319)}{-1.063}\Delta rd_{t-1} \underset{(-2.021)}{-1.323}\Delta rd_{t-2} \underset{(-2.815)}{-2.243}\Delta rd_{t-4} \underset{(-3.542)}{-2.922}\Delta rd_{t-5} \underset{(-2.514)}{-1.754}\Delta rd_{t-6} \underset{(-3.127)}{-1.576}\Delta rd_{t-7}$$
$$\underset{(-5.425)}{-1.382}\Delta rd_{t-8} \underset{(-3.756)}{-0.913}\Delta rd_{t-10} + \underset{(3.183)}{8.822}\Delta ccad_{t-1} + \underset{(3.502)}{9.204}\Delta ccad_{t-2} + \underset{(3.656)}{8.649}\Delta ccad_{t-3} +$$
$$\underset{(3.986)}{8.395}\Delta ccad_{t-4} + \underset{(4.294)}{7.993}\Delta ccad_{t-5} + \underset{(4.367)}{6.807}\Delta ccad_{t-6} + \underset{(4.348)}{5.278}\Delta ccad_{t-7} + \underset{(3.837)}{3.430}\Delta ccad_{t-8}$$
$$+ \underset{(3.401)}{2.064}\Delta ccad_{t-9} + \underset{(2.441)}{0.783}\Delta ccad_{t-10} + \underset{(2770)}{0.260}\phi_{t-1}$$

As be shown in the above equation, the coefficient of error correction term, λ, is positive and statistically significant. Second, the existence of λ further support the conclusion that co-integration exists among the variables as shown in the previous part. Third, the coefficient of error correction term indicates that approximate 0.26 of the change in the real exchange rate per quarter can be attributed to the disequilibrium between actual and equilibrium levels. Fourth, the error correction model indicates that the change in the expected interest rate differentials and cumulated current account have a short run effect on the real exchange rate besides a long run effect.

5.4 Model checking

If the model defects such as residual autocorrelation or ARCH effects are detected at

[14] See jyh-lin wu 1999

the checking stage, this is usually regards as an indication that the model is a poor representation of the DGP. Now I make some diagnostic test for autocorrelation, heteroscedasticity, normality and structural stability. LM test will be used for autocorrelation; tests for non-normality can be used by Jarque-Bera tests; ARCH-LM test is used to test heteroscedasticity ; CUSUM tests will be used to test stability of the model. As be shown in table 4, other LM tests indicate that there is existence of autocorrelation with 5% significance level except for LM (1). Autocorrelation leads to an upward bias in estimates of the statistical significance of coefficient estimates which basically means that there are better coefficients to include in the model. JARQUE=BERA test shows that residuals are normal distribution. ARCH-LM indicates that there is no arch effect on residuals. It means that the residual has a constant and invariant time variance. Finally chow test shows the model is stable.

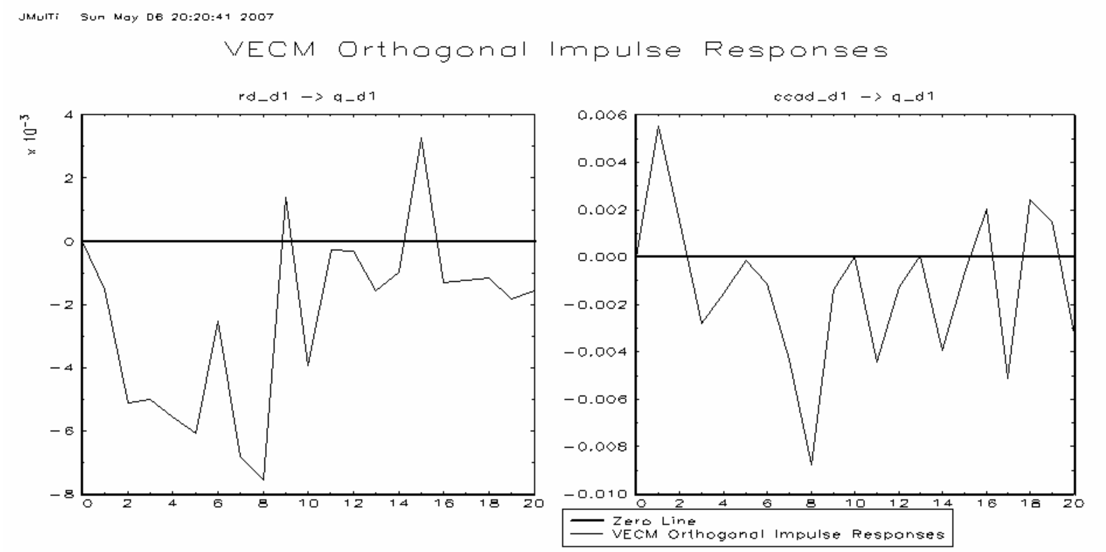
Table 4

LM-TYPE TEST	**JARQUE-BERA TEST**	**ARCH-LM TEST**	**CHOW TEST**
LM(1)=0.2346 **LM(2)=0.0042** **LM(3)=0.0046** **LM(4)=0.0002**	**P-Value(Chi^2)=0.9665** **Skewness=0.0104** **Kurtosis=2.8083**	**ARCH(1)=0.4553** **ARCH(2)=0.2490** **ARCH(3)=0.5150** **ARCH(4)=0.2867**	**Forecast test = 0.1715** **Bootstrapped p-value = 0.93** **Asymptotic F p-value = 0.9984**

5.5 Impulse response analysis

Impulse response analysis shows the reaction or response of one variable against the shock happening for another variable. Namely, it reveals the reaction or adjustment of one variable that is due to a sudden change of another variable. From the below graph, I found that a shock in interest rate differentials and accumulated current account have significant effect on movement of real exchange rate because a movement is observable from the graph.

Graph 4



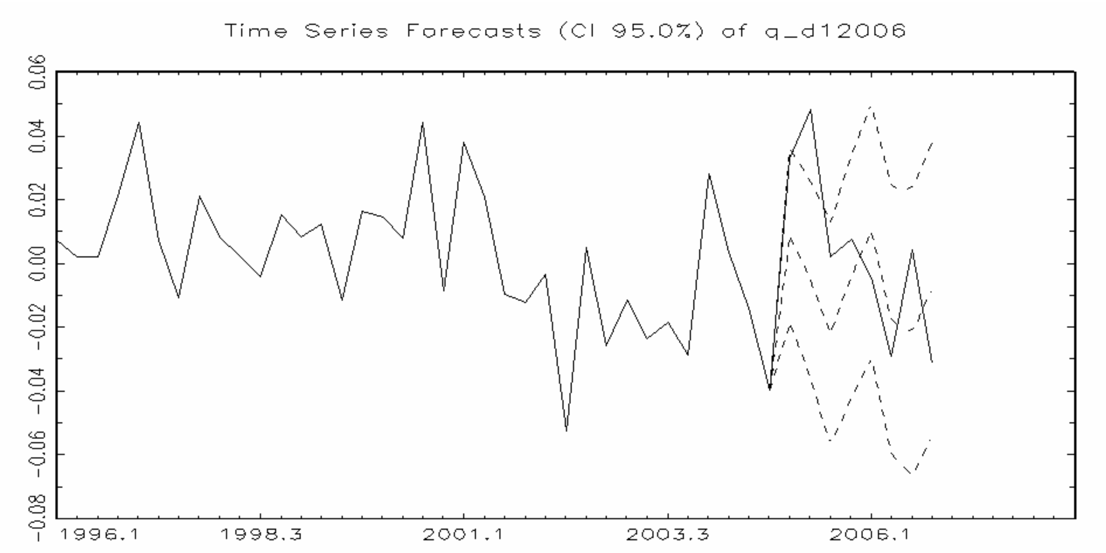
5.6 Forecast

Forecast is very important step in economics time series analysis. First, I apply the estimated model to predict change in real exchange rate from 2007 Q1 up to 2009 Q2. The results are shown in table 8 of appendix. In addition to predict the time series in future, forecast can be used to check the goodness of the model. I have re-estimated the model using the data from 1993 Q1 to 2004 Q4, and use that model to predict change in real exchange rate for next 8 quarters. Most of time series lie within 95% CI coverage except for data in 2005 Q2. Although data in 2005 Q2 is outside interval, I found that real data and predicted data follow same direction, and the truth can be confirmed by the table 6 and graph 5. Therefore, the check indicates the adequacy of the model.

Table 6

Time		Lower CI	Forecast	Real data	Upper CI		+/-
2005 Q1	-0.0457	-0.0184	0.0088	0.0328578	0.0361	0.0634	0.0273
2005 Q2	-0.0669	-0.036	-0.0052	0.0480764	0.0257	0.0566	0.0309
2005 Q3	-0.0917	-0.0568	-0.022	0.0022125	0.0129	0.0478	0.0349
2005 Q4	-0.0806	-0.0425	-0.0044	0.0075823	0.0337	0.0718	0.0381
2006 Q1	-0.0695	-0.0296	0.0102	-0.004666	0.0501	0.09	0.0399
2006 Q2	-0.1021	-0.0602	-0.0184	-0.029064	0.0235	0.0654	0.0419
2006 Q3	-0.1113	-0.0665	-0.0217	0.0043343	0.0231	0.0679	0.0448
2006 Q4	-0.1003	-0.054	-0.0077	-0.031385	0.0386	0.0849	0.0463

Graph 5


Time Series Forecasts (CI 95.0%) of q_d12006
0.06
0.04
0.02
0.00
-0.02
-0.04
-0.06
-0.08
1996.1
1998.3
2001.1
2003.3
2006.1

6. Conclusions and summary

The question the paper asks is: is there a long-run relationship between real exchange rate and real interest differentials? And if so, what empirical representation of it does the data support? Many investigations, such as Meese and Rogoff (1998) and Edison and Pauls (1993), did not find a long run relationship between the real exchange and interest rate differentials. In this paper, I re-examined the relationship between real exchange rates and expected real interest differentials over the period of recent floating exchange rates. I have found some interesting things.

First, I found that the original data are not stationary by using unit root test, but the data became stationary after being took first difference. The result is consistent with those of previous papers (Coughlin and Koedijk (1990), Blundell-Wignall and Browne (1991), Meese and Rogoff (1988) and Edison and Pauls) I mentioned in introduction part. Second, using Johansen co-integration method, I found that the long-run relationship between real exchange rates and real interest differentials exists when cumulated current accounts are included. The result is consistent with those of Coughlin and Koedijk (1990) and Blundell-Wignall and Browne (1991), but in contrast to those of Meese and Rogoff (1988) and Edison and Pauls (1993). Third, using model checking, I found that there is no arch-effect, the residuals are normality and the model is stable, whereas there is autocorrelation. By comparing the predicted data and the real ones, Final conclusion is that the empirical evidence is provided to show that our error-correction model leads to a good real exchange rate forecast. Although I follow the same strategy as Jyh-Lin Wu, he used root-mean-square error (RMSE) and mean-absolute error (MAE) to compare error correction model and random model and concluded that forecasting accuracy of ECM outperforms that of a random walk model in out of sample forecasts.

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Appendix

Figure1

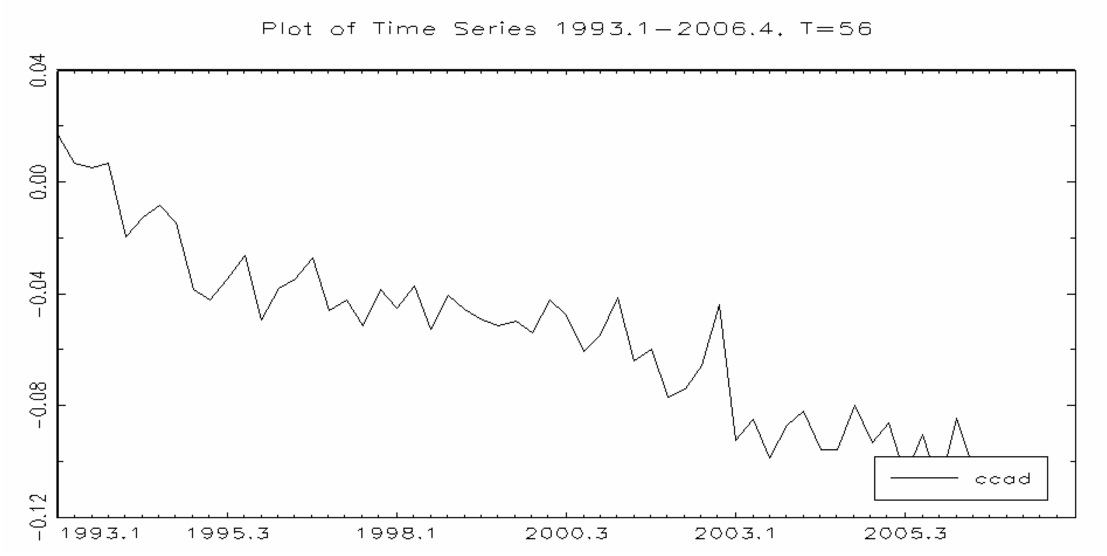

Figure 2

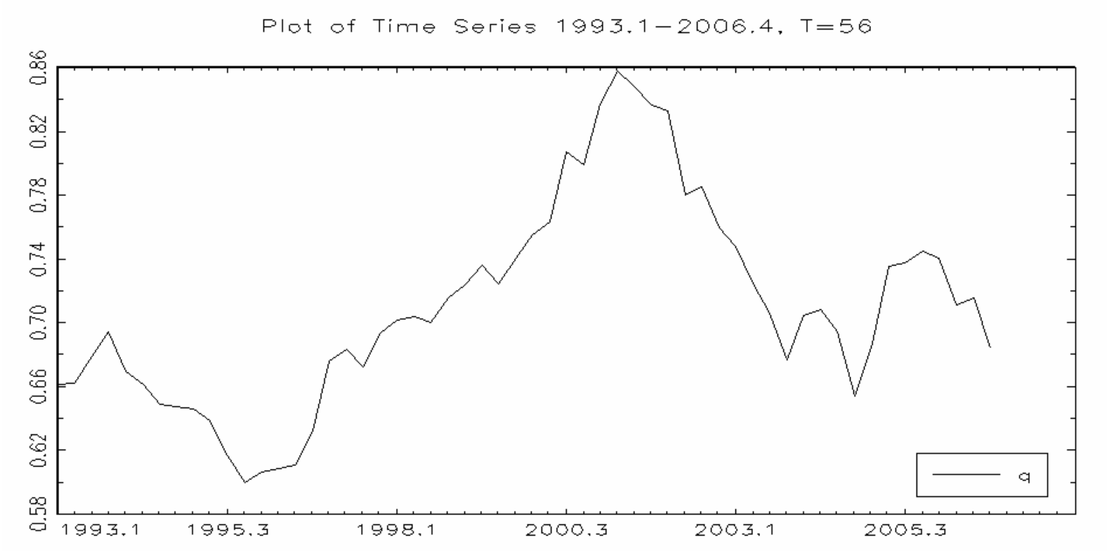

Figure 3

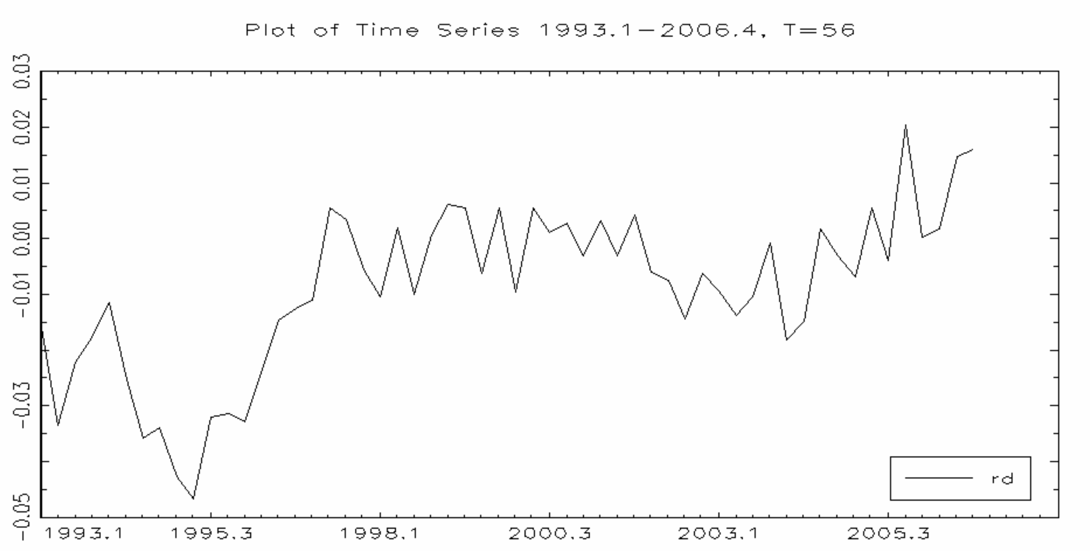

Figure 4

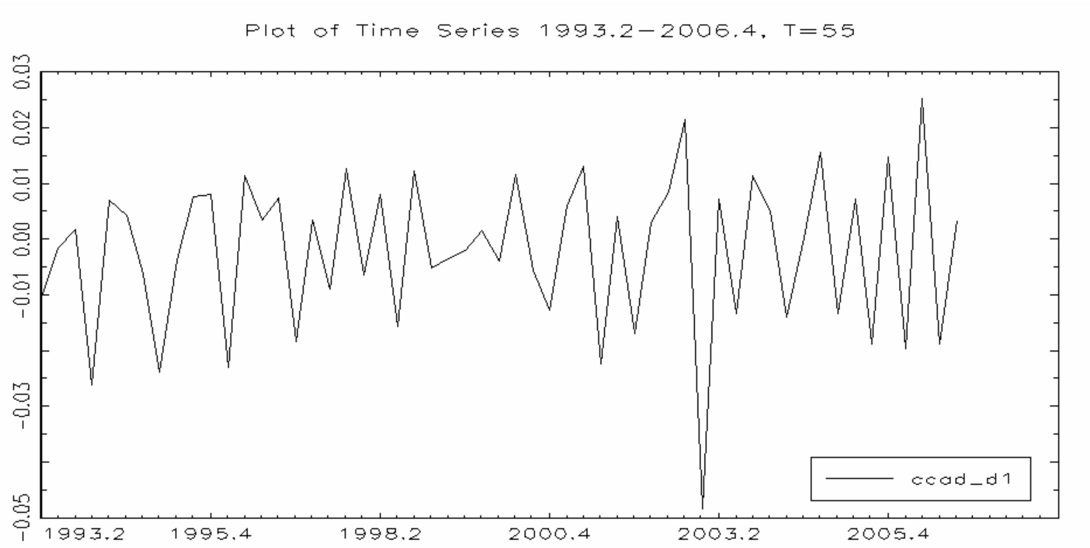

Figure 5

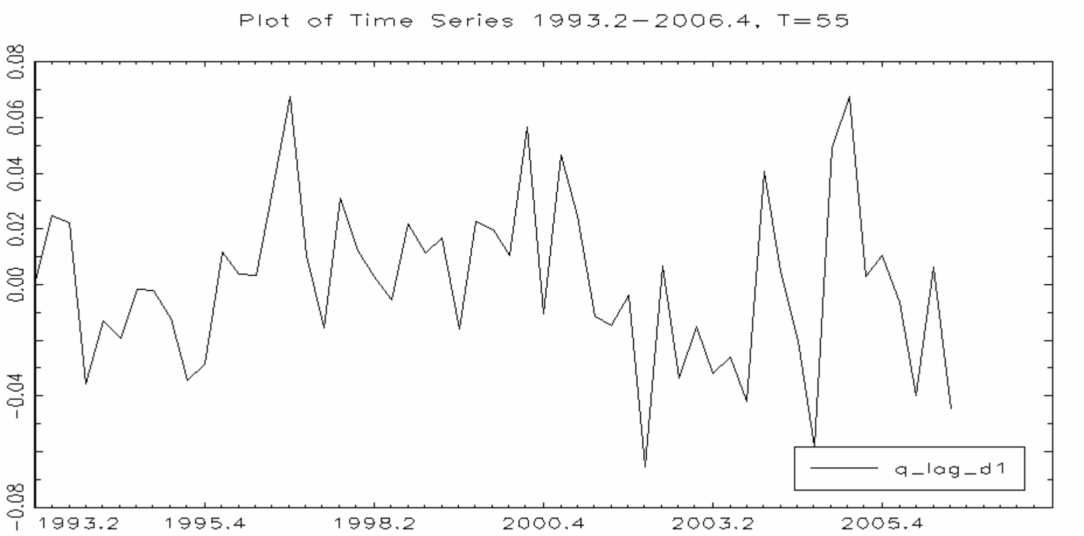

Figure 6

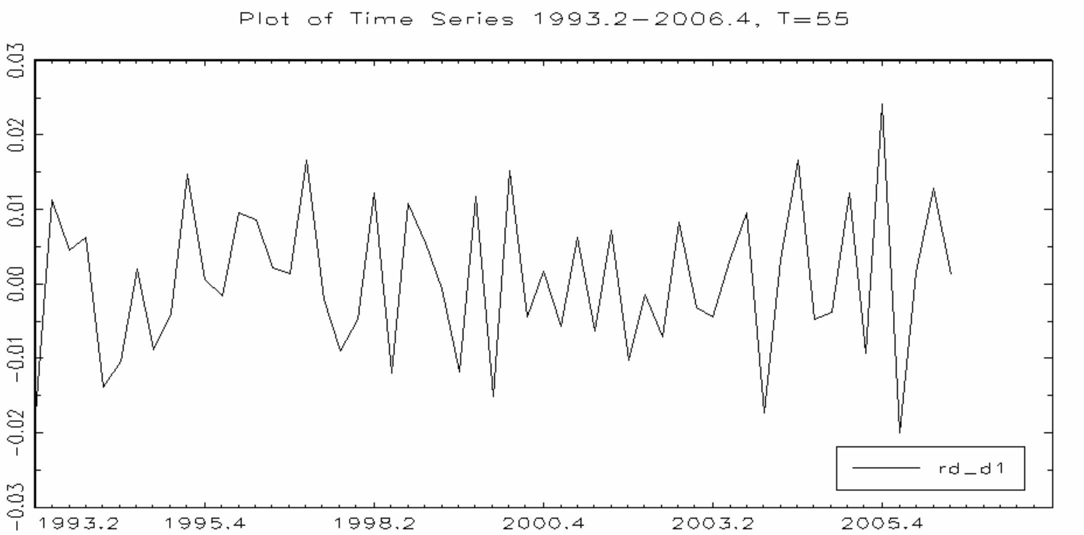

Figure 7

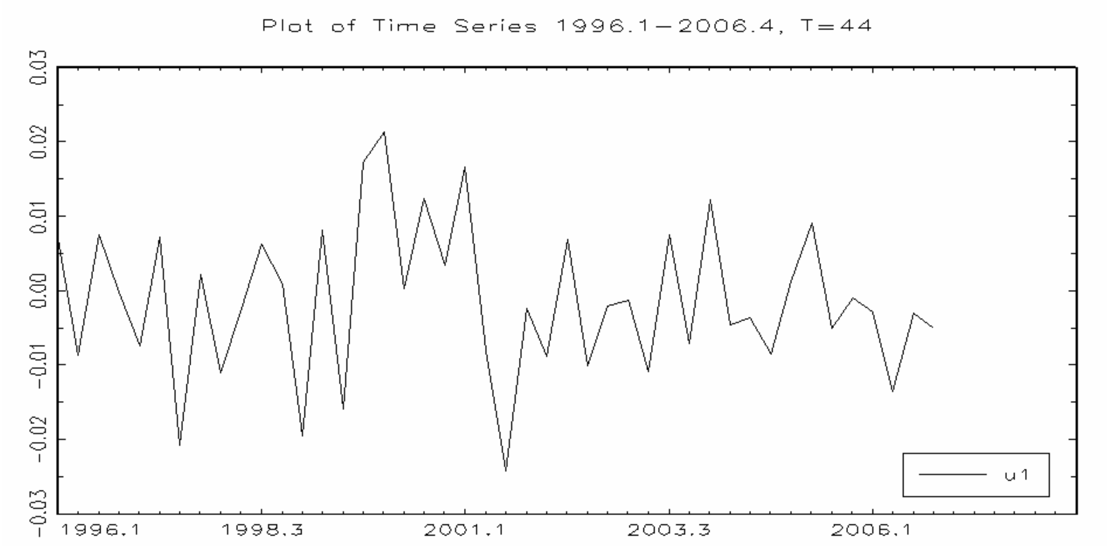

Figure 8

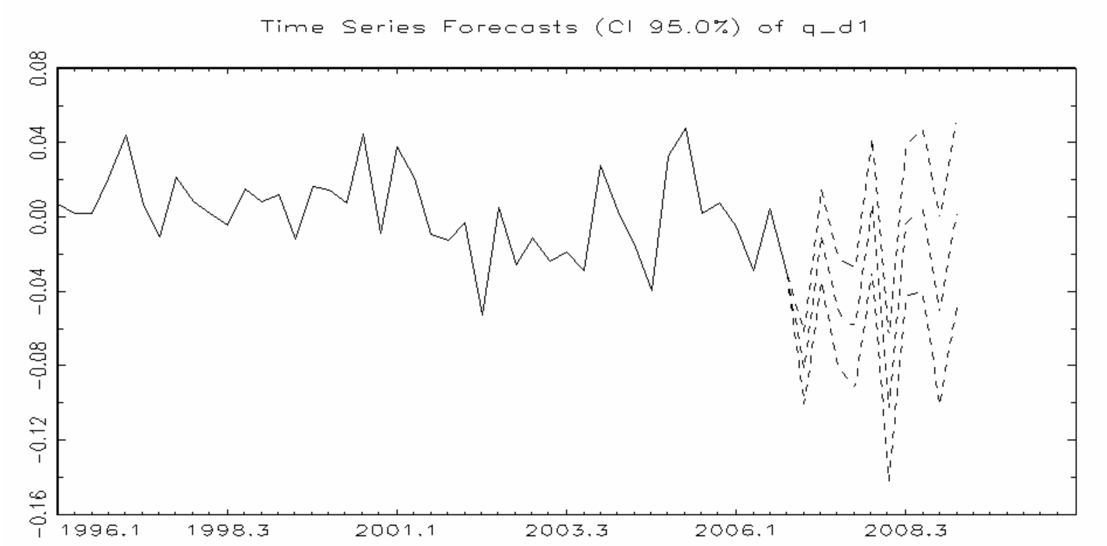

Table 1: test statistics

DESCRIPTIVE STATISTICS:

variable	mean	min	max	std. dev.
ccad	-5.42212e-02	-1.09817e-01	1.69510e-02	3.09467e-02
q	7.10417e-01	5.99343e-01	8.57881e-01	6.33655e-02
rd	-8.94942e-03	-4.66593e-02	2.03456e-02	1.45896e-02

JARQUE-BERA TEST

variable	teststat	p-Value(Chi^2)	skewness	kurtosis
ccad	0.9366	0.6261	0.1659	2.4603
q	2.2610	0.3229	0.4780	2.7655
rd	3.3555	0.1868	-0.5992	2.9574

ARCH-LM TEST with 2 lags

variable	teststat	p-Value(Chi^2)	F stat	p-Value(F)
ccad	33.6877	0.0000	44.7793	0.0000
q	42.4426	0.0000	99.1527	0.0000
rd	20.6206	0.0000	16.6797	0.0000

Table2

	d(q_d1)	d(rd_d1)	d(ccad_d1)
d(q_d1) (t-1)	-0.841	-0.107	-0.093
	(0.156)	(0.057)	(0.097)
	{0.000}	{0.061}	{0.342}
	[-5.384]	[-1.877]	[-0.950]
d(rd_d1) (t-1)	-1.132	-1.063	-0.691
	(0.459)	(0.167)	(0.286)
	{0.014}	{0.000}	{0.016}
	[-2.466]	[-6.372]	[-2.412]
d(ccad_d1)(t-1)	10.291	1.763	4.744
	(2.994)	(1.088)	(1.868)
	{0.001}	{0.105}	{0.011}
	[3.437]	[1.620]	[2.540]
d(q_d1) (t-2)	-0.660	0.147	0.033
	(0.174)	(0.063)	(0.108)
	{0.000}	{0.020}	{0.762}
	[-3.800]	[2.325]	[0.303]
d(rd_d1) (t-2)	-1.708	-1.229	-0.854
	(0.763)	(0.277)	(0.476)
	{0.025}	{0.000}	0.073}
	[-2.239]	[-4.435]	[-1.796]
d(ccad_d1)(t-2)	10.769	1.671	3.881
	(2.884)	(1.048)	(1.799)
	{0.000}	{0.111}	{0.031}
	[3.734]	[1.595]	[2.158]
d(q_d1) (t-3)	-0.455	0.013	0.207
	(0.163)	(0.059)	(0.102)
	{0.005}	{0.827}	{0.042}
	[-2.787]	[0.219]	[2.032]
d(rd_d1) (t-3)	-1.848	-1.025	-1.060
	(0.962)	(0.350)	(0.600)
	{0.055}	{0.003}	{0.078}
	[-1.920]	[-2.930]	[-1.765]
d(ccad_d1)(t-3)	10.250	1.570	2.888
	(2.662)	(0.967)	(1.660)
	{0.000}	{0.105}	{0.082}
	[3.850]	[1.623]	[1.739]

Continuous

	d(q_d1)	d(rd_d1)	d(ccad_d1)
d(q_d1)(t-4)	-0.295	-0.096	0.014
	(0.154)	(0.056)	(0.096)
	{0.055}	{0.087}	{0.886}
	[-1.917]	[-1.713]	[0.143]
d(rd_d1) (t-4)	-2.503	-0.758	-0.740
	(0.993)	(0.361)	(0.619)
	{0.012}	{0.036}	{0.232}
	[-2.521]	[-2.101]	[-1.194]
d(ccad_d1)(t-4)	9.918	1.443	2.131
	(2.413)	(0.877)	(1.505)
	{0.000}	{0.100}	{0.157}
	[4.111]	[1.646]	[1.416]
d(q_d1) (t-5)	-0.127	-0.077	0.121
	(0.151)	(0.055)	(0.094)
	{0.401}	{0.162}	{0.198}
	[-0.840]	[-1.399]	[1.288]
d(rd_d1) (t-5)	-3.192	-0.694	-0.694
	(1.025)	(0.373)	(0.640)
	{0.002}	{0.063}	{0.278}
	[-3.113]	[-1.862]	[-1.086]
d(ccad_d1) (t-5)	9.463	1.154	1.591
	(2.184)	(0.793)	(1.362)
	{0.000}	{0.146}	{0.243}
	[4.333]	[1.454]	[1.168]
d(q_d1) (t-6)	-0.157	0.026	0.179
	(0.160)	(0.058)	(0.100)
	{0.325}	{0.654}	{0.073}
	[-0.983]	[0.448]	[1.792]
d(rd_d1) (t-6)	-1.993	-0.212	-0.418
	(0.855)	(0.311)	(0.533)
	{0.020}	{0.494}	{0.433}
	[-2.331]	[-0.684	[-0.784]
d(ccad_d1)(t-6)	8.168	0.705	0.968
	(1.875)	(0.681)	(1.169)
	{0.000}	{0.301}	{0.408}
	[4.357]	[1.035]	[0.828]

Continuous

	d(q_d1)	d(rd_d1)	d(ccad_d1)
d(q_d1) (t-7)	-0.389	-0.001	0.175
	(0.159)	(0.058)	(0.099)
	{0.015}	{0.991}	{0.078}
	[-2.444]	[-0.011]	[1.763]
d(rd_d1) (t-7)	-1.685	-0.165	-0.361
	(0.674)	(0.245)	(0.421)
	{0.012}	{0.500}	{0.390}
	[-2.499]	[-0.674]	[-0.859]
d(ccad_d1)(t-7)	6.374	0.435	0.272
	(1.478)	(0.537)	(0.922)
	{0.000}	{0.418}	{0.768}
	[4.313]	[0.810]	[0.295]
d(q_d1) (t-8)	-0.352	-0.057	0.087
	(0.164)	(0.060)	(0.102)
	{0.032}	{0.341}	{0.395}
	[-2.149]	[-0.953]	[0.851]
d(rd_d1) (t-8)	-1.397	-0.063	-0.663
	(0.526)	(0.191)	(0.328)
	{0.008}	{0.741}	{0.043}
	[-2.656]	[-0.330]	[-2.020]
d(ccad_d1)(t-8)	4.212	0.047	-0.185
	(1.086)	(0.395)	(0.678)
	{0.000}	{0.904}	{0.784}
	[3.877]	[0.120]	[-0.273]
d(q_d1) (t-9)	-0.037	-0.047	0.196
	(0.151)	(0.055)	(0.094)
	{0.807}	{0.395}	{0.037}
	[-0.245	[-0.851]	[2.088]
d(rd_d1) (t-9)	-0.024	-0.160	-0.324
	(0.413)	(0.150)	(0.257)
	{0.953}	{0.287}	{0.208}
	-0.059]	[-1.065]	[-1.259]
d(ccad_d1)(t-9)	2.530	-0.220	-0.287
	(0.716)	(0.260)	(0.447)
	{0.000}	{0.398}	{0.521}
	[3.532]	[-0.846]	[-0.642]

Continuous

	d(q_d1)	d(rd_d1)	d(ccad_d1)
d(q_d1) (t-10)	-0.366	-0.015	0.040
	(0.136)	(0.050)	(0.085)
	{0.007}	{0.766}	{0.640}
	[-2.682]	[-0.297]	[0.467]
d(rd_d1) (t-10)	-0.972	0.049	-0.319
	(0.331)	(0.120)	(0.206)
	{0.003}	{0.682}	{0.122}
	[-2.937]	[0.410]	[-1.544]
d(ccad_d1)(t-10)	0.895	0.117	-0.173
	(0.333)	(0.121)	(0.208)
	{0.007}	{0.335}	{0.405}
	[2.688]	[0.964]	[-0.833]

Table 3

LM-TYPE TEST FOR AUTOCORRELATION with 1 lags

LM statistic:	11.6359
p-value:	0.2346
df:	9.0000

JARQUE-BERA TEST

variable	teststat	p-Value(Chi^2)	skewness	kurtosis
u1	0.0682	0.9665	0.0104	2.8083
u2	1.7599	0.4148	0.2932	2.2151
u3	0.3966	0.8201	-0.1853	2.7190

MULTIVARIATE ARCH-LM TEST with 1 lags

VARCHLM test statistic:	36.2869
p-value(chi^2):	0.4553
degrees of freedom:	36.0000

Table 4

LM-TYPE TEST FOR AUTOCORRELATION with 2 lags

LM statistic: 37.7669
p-value: 0.0042
df: 18.0000

JARQUE-BERA TEST

variable	teststat	p-Value(Chi^2)	skewness	kurtosis
u1	0.0682	0.9665	0.0104	2.8083
u2	1.7599	0.4148	0.2932	2.2151
u3	0.3966	0.8201	-0.1853	2.7190

MULTIVARIATE ARCH-LM TEST with 2 lags

VARCHLM test statistic: 79.7324
p-value(chi^2): 0.2490
degrees of freedom: 72.0000

Table 5

LM-TYPE TEST FOR AUTOCORRELATION with 3 lags
LM statistic: 49.9782
p-value: 0.0046
df: 27.0000 JARQUE-BERA TEST

variable	teststat	p-Value(Chi^2)	skewness	kurtosis
u1	0.0682	0.9665	0.0104	2.8083
u2	1.7599	0.4148	0.2932	2.2151
u3	0.3966	0.8201	-0.1853	2.7190

MULTIVARIATE ARCH-LM TEST with 3 lags

VARCHLM test statistic: 106.7847
p-value(chi^2): 0.5150
degrees of freedom: 108.00

Table 6

LM-TYPE TEST FOR AUTOCORRELATION with 4 lags

LM statistic:	**73.3319**
p-value:	**0.0002**
df:	**36.0000**

JARQUE-BERA TEST

variable	teststat	p-Value(Chi^2)	skewness	kurtosis
u1	0.0682	0.9665	0.0104	2.8083
u2	1.7599	0.4148	0.2932	2.2151
u3	0.3966	0.8201	-0.1853	2.7190

MULTIVARIATE ARCH-LM TEST with 4 lags

VARCHLM test statistic:	**153.0771**
p-value(chi^2):	**0.2867**
degrees of freedom:	**144.0000**

Table 7

CHOW TEST FOR STRUCTURAL BREAK
On the reliability of Chow-type tests..., B. Candelon, H. Lütkepohl, Economic Letters 73 (2001), 155-160

sample range: **[1996 Q1, 2006 Q4], T = 44**
tested break date: **2005 Q3 (38 observations before break)**

break point Chow test not possible for given break date
sample split Chow test not possible for given break date
Chow forecast test: **0.1715**
: **0.9300**
(F p-value not valid with subset restrictions)
asymptotic F p-value: **0.9984**
degrees of freedom: **18, 6**

Table 8

time	forecast	lower CI	upper CI	+/-
2007 Q1	-0.0815	-0.1009	-0.0620	0.0195
2007 Q2	-0.0100	-0.0347	0.0147	0.0247
2007 Q3	-0.0516	-0.0806	-0.0226	0.0290
2007 Q4	-0.0595	-0.0919	-0.0271	0.0324
2008 Q1	0.0066	-0.0289	0.0422	0.0355
2008 Q2	-0.1028	-0.1419	-0.0638	0.0390
2008 Q3	-0.0018	-0.0426	0.0390	0.0408
2008 Q4	0.0037	-0.0402	0.0477	0.0439
2009 Q1	-0.0513	-0.1012	-0.0014	0.0499
2009 Q2	0.0029	-0.0473	0.0531	0.0502