

MASTER DEGREE PROJECT

**EVALUATION OF SINGLE AND
THREE FACTOR CAPM BASED ON
MONTE CARLO SIMULATION**

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ABSTRACT

The aim of this master thesis was to examine whether the noticed effect of Black Monday October 1987 on stock market volatility has also influenced the predictive power of the single factor CAPM and the Fama French three factor CAPM, in order to conclude whether the models are less effective after the stock market crash. I have used an OLS regression analysis and a Monte Carlo Simulation technique. I have applied these techniques on 12 industry portfolios with US data to draw a conclusion whether the predictability of the single and three factor model has changed after October 1987. My research confirms that the single factor CAPM performs better before October 1987 and also found evidences that support the same hypothesis of Black Monday effect on the predictive power of the Fama French three factor model.

Key words: Single factor CAPM, Fama French CAPM, OLS regression, Monte Carlo Simulation.

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Abbreviations

MPT	Modern Portfolio Theory
CAPM	Capital Asset Pricing Model
SMB	Small Minus Big
HML	High Minus Low
OLS	Ordinary Least Squares
MCS	Monte Carlo Simulation
NYSE	New York Stock Exchange
AMEX	American Stock and Options Exchange
NASDAQ	National Association of Securities Dealers Automated Quotations
CRSP	Center for Research in Securities Prices
ME	Market Equity
BE/ME	Book Equity to Market Equity
MAV	Mean Absolute Values

1 Introduction

The relationship between risk and return has long been a centerpiece in the field of finance. Since William Sharpe (1964) and John Lintner (1965) found a linear relationship between expected returns of assets and their market betas and developed the famous Capital Asset Pricing Model (CAPM), the debate over the testability, validity and predictive power of CAPM has not ceased. Early studies of CAPM by Black, Jensen, Scholes (1972) and Fama and MacBeth (1973) found empirical support of the CAPM, but shortly after that Roll (1977) questioned the testability of CAPM and additional studies of the model discovered some anomalies. Banz (1981) discovered one of the main empirical failures of CAPM - the “size effect” and Fama and French (1992, 1993) found that not only the size of the firm but also book-to-market ratio is related to the returns and include them as an explanatory variables in addition to the market beta in their three factor model. Black (1993) cast doubt on the Fama French model arguing that “just after the small-firm effect was announced, it seems to have vanished”¹. Kothari, Shanken, and Sloan (1995) also questioned the Fama French model, claiming that the findings of Fama and French depend on the interpretation of the statistical tests and provided evidence for the existence of sample selection biases. Further studies of the model Harvey (1989), Ferson and Harvey (1991, 1993), and Ferson and Korajczyk (1995) found that betas similar to expected returns are time varying. Jagannathan and Wang (1993) questioned some of the CAPM assumptions, and in other studies non risk based factors were claimed to have explanatory power.

Indisputably, the single and Fama French three factor CAPM have been tested, studied and debated a lot over the years. There can be found both proponents and opponents of the models and a lot of extensions and modifications of both models have been developed. What provoked the interest in studying again these models is a pattern observed only after the October 1987 crash and described by Mark Rubinstein (1994) in his article “Implied Binomial Trees”. In October 19, 1987 the Dow Jones Industrial Average fell sharply by 22,61% and caused enormous drops in stock markets across the world. After that it has been noticed that the volatility of equity is a decreasing function of price; with the decrease of a company’s equity in value, the company’s leverage increases, which results in an increased volatility of the equity and makes even lower stock prices more likely to occur, and vice versa. Rubinstein relates this pattern to the market trader’s behavior and introduces the term “crashophobia”. In 1997 G. William Schwere in his study “Stock Market Volatility: Ten Years After the Crash” concludes “that the volatility associated with the 1987 crash was brief and transitory” and since then it has been low and stable. Since option prices contain unique information about the market traders’ assessment of the price process of the underlying assets, information that is more comprehensive than the information contained in a time series of stock returns, and since there is an empirical support of the crashophobia but a study extended to the effect of the stock market crash on the CAPM effectiveness has not been found, the master thesis aim and contribution will be to find whether the changes in volatility of the equity observed after the stock market crash of October 1987 has also caused changes in the predictive power of the single and three factor CAPM.

¹ Black, Fischer. "Estimating Expected Return." Financial Analysts Journal, September/October 1993

The master thesis focuses on those two capital asset pricing models because, as James Davis says in his article “Reviewing the CAPM” on the 2006 Risk Management Conference, CAPM “is the most well-known asset-pricing model” and “the use of CAPM is a favorite because it is a model with only one risk factor, the underlying logic is powerful, and it is well known and widely understood” and the Fama French model is “perhaps the most promising alternative” and “the most widely used model of stock returns in the academic finance literature”. Furthermore, the master thesis concentrates on the US market and all the tests will be applied to the US data, because to test the effect of the stock market crash of October 1987 on the single and three factor CAPM it is necessary both models to hold and both models have been tested and proved to work for the US market.

The master thesis starts with presenting the main empirical studies that support and challenge the single and three factor CAPM. It continues with the assumptions and implications of the capital asset pricing model and describes the single and three factor CAPM. A description of the data used, the selection methods and argumentations of these decisions follow. The thesis continues with specifying the econometric methods that are applied to the data – ordinary least-squares regression and a Monte Carlo Simulation technique. Next the hypothesis whether the stock market crash of October 1987 has influenced the predictive power of the single and three factor CAPM is tested by applying US data on the specified capital asset pricing models by means of the specified methods, namely OLS regression and Monte Carlo Simulation. Then the results obtained are put forward, analyzed and a conclusion is drawn.

2 Capital Asset Pricing Model (CAPM)

The aim of this section is to introduce the main empirical studies that support and challenge the single and three factor CAPM; to present the assumptions and implications of the capital asset pricing models and to describe in detail the logic of both models.

Key words: single factor CAPM, alpha, beta, sigma, three factor CAPM, SMB, HML.

2.1 Studies of the CAPM

2.1.1 Studies in support of CAPM

One of the assumptions of CAPM is that there is a risk-free asset and investors can borrow and lend at a fixed risk free rate over the investment horizon. If, however, they cannot borrow and lend at the risk free rate, they can pick any portfolio that is on the efficient frontier according to the risk they are willing to take. Thus the market portfolio may not be mean variance efficient any longer; hence the relationship between the expected return and beta as defined by CAPM may not characterize the market equilibrium.

In 1972, Fisher Black constructed a model that represents the expected return of any asset as a linear function of the expected return of any two frontier portfolios, providing the following characteristics hold: firstly, the combination of efficient portfolios will give a portfolio that also lies on the efficient frontier, and secondly, assuming that the efficient frontier is divided

in two parts efficient and inefficient, every portfolio from the efficient part has a companion portfolio, a zero beta portfolio, situated on the inefficient part and there is no correlation between them.

Black's model of CAPM is in the case of absence of a risk free asset, when there is risk free lending but not borrowing and when there is a borrowing but at a rate that is higher than the risk free rate.

In 1972 Black, Jensen, and Scholes conducted a study of all the stocks on the NYSE. Using 1931-1965 data, they formed portfolios and regressed them on beta. They found that the relationship between the average portfolio return and beta is close to linear and that the data are consistent with Black's (1972) model of the CAPM.

One year later, Fama and MacBeth (1973) carried out a study of the stocks traded on the NYSE, covering the period from 1926 to 1968. The conclusion of the Fama and MacBeth study was that the data generally support the CAPM. They found that the intercept term is larger than the risk-free rate, that there is a linear relationship between the average portfolio return and beta and that the linear relationship remains strong for a long time period.

2.1.2 Studies that challenge CAPM

In 1977 Roll questioned the testability of CAPM, his main critique being that the CAPM cannot be tested or applied until the structure of the true market portfolio is known and all securities are included. Using a proxy incurs two problems, namely the proxy might be efficient when the true market portfolio is not and the reverse, the proxy might not be efficient when the market portfolio is. Furthermore, there is a possibility of benchmark error as using different proxies yields different results and conclusions and inappropriate proxy might be taken. In addition, in reality, the return on the market portfolio is unobservable as in general it can include non traded financial assets such as consumer durables, real estate, and human capital, and international stocks and bonds.

Another Roll's critique of CAPM is the use of ex post data, which is only an approximation. Taking the historical average returns might lead to imprecise inferences about expected returns as it is very likely that returns are time varying or in other words that they do not remain constant during longer periods. This can be explained with the new information that occurs at different rate hence the revisions of investors' assessments are time varying.

Usually in the CAPM nominal excess returns are used as proxies for real excess returns. Provided we assume that the nominal risk free rate includes also the inflation premium then we use nominal prices and treat the excess returns as real. Still the historical returns might not be good proxies for future expected returns.

In the early 1980s other studies of CAPM started to challenge its predictive power by suggesting that other factors except from CAPM's beta can influence the relationship between risk and return and explain the residual variation in the average returns. Banz (1981) was the first to notice the "size effect" or that size and return are related and that over long periods of time, small firms outperform large firms. Banz studied firms on the NYSE during

the period 1936-1975 and estimated the cross-sectional relation between the average returns, beta and the relative size of firms. He found that the average returns on stocks of large firms were smaller than those of stocks of small firms and concluded that there is large and statistically significant size effect.

The study of CAPM conducted by Fama and French in 1992 confirmed Banz's finding and went further by including the firm's book-to-market ratio as an explanatory variable. They showed that both size effect and value effect can explain a great part of the variations in average returns and even that book-to-market ratio can have stronger explanatory power than size. After that many multifactor models were examined in order to capture deviations from the CAPM.

2.1.3 Further Studies of CAPM and Fama French three factor model

In 1995 Kothari, Shanken, and Sloan claimed that the findings of Fama and French depend on the interpretation of the statistical tests. As long as CAPM is based on ex post data, a group of risk factors that can approximate the intercept to zero can always be found but the explanation of deviations might be far more complicated and non risk based factors could be involved as well. When relying on ex post data, data-snooping biases and selection biases add to the difficulty of quantifying deviations.

Breen and Korajczyk (1993) and Kothari, Shanken, and Sloan (1995) provided evidence for the existence of sample selection biases and Campbell, Lo and MacKinlay (1997) for the data-snooping biases and stated that "Data-snooping biases refer to the biases in statistical inference that result from using information from data to guide subsequent research with the same or related data"¹. As far as tests are based on ex post data, by grouping assets that has common disturbance terms and examining and re-examining them, sooner or later variables that can explain and predict returns will be found. In line with the said above, Fischer Black (1993) commented that "just after the small-firm effect was announced, it seems to have vanished."²

Several studies questioned the validity of beta; more specifically they claimed that beta calculated on the basis of historical data might not predict well the variance of future returns. Harvey (1989), Ferson and Harvey (1991, 1993), and Ferson and Korajczyk (1995) found that betas similar to expected returns are time varying.

As every model CAPM tries to explain a real system in a similar but simpler structure providing different assumptions are made. But some of those assumptions are not realistic and make CAPM difficult to implement. For example, investors not always agree on the return, risk and correlation of the assets and even if they agree, some of them might be constrained from investing in certain classes of assets. Furthermore, investors can have different time horizons and hence they will consider different assets as risk free. Investors can impose leverage constraints and in order to leverage or vice versa they can decide on different portfolios. Another assumption that can be questioned is that investors are mean variance optimizers, as in reality many might not be.

¹ <http://data-snooping.martinsewell.com/>

² Black, Fischer. "Estimating Expected Return." *Financial Analysts Journal*, September/October 1993

Jagannathan and Wang (1993) questioned some of the CAPM assumptions and suggested that instead of the return on broad stock market indexes, a return on the aggregate wealth portfolio of all agents in the economy should be taken. Jagannathan and Wang included human capital in their measure of wealth, using growth of labor income as a proxy for the human capital. For their study, they created a multiple beta model of CAPM and allowed for time varying beta and returns. The conclusion that Jagannathan and Wang reached is that by taking the return on the aggregate wealth portfolio and allowing for time varying beta and returns, the CAPM is able to explain more than fifty percent of the cross-sectional variation in average returns.

It is possible to find other non risk based factors that have explanatory power. Irrational behaviour of market participants was suggested by DeBondt and Thaler (1985) as one of the reasons for the CAPM deviations. In addition, Lakonishok, Shleifer, and Vishny (1994) pointed out that some investors assume trends in asset prices, extrapolate past growth rates for far too long future periods, and respond too strongly to good or bad news, which again can lead to poor performance of the CAPM. Amihud and Mendelson (1986) found that market frictions and demands for liquidity explain some deviations of CAPM.

2.2 CAPM Assumptions

CAPM assumes that capital markets are efficient, all securities and assets are correctly priced and there are no arbitrage opportunities.

Investors are risk-averse and mean-variance optimizers. For a specific level of risk they will prefer higher returns and for a specific expected return they will prefer lower risk. The choice of assets is based only on the risk preference and preferences toward markets or assets are excluded.

The market portfolio is assumed to consist of all assets in all markets, but many assets such as real estate, human capital and others are not included. Thus the market portfolio is a limited version of the real market portfolio.

Another assumption of CAPM is that there are many investors and all investors are price takers i.e. their transactions have no effect on the market.

Investors plan to invest over the same time horizon and are interested in only one period ahead. They make their investment decisions at the beginning of the period and there are not any changes during the investment horizon.

There is a risk-free asset that pays interest rate r_f in zero net supply. Investors can borrow and lend at a fixed risk free rate over the investment horizon.

There are no taxes on returns or transactions costs such as commissions, service charges. In reality, the income from interest, dividends or capital gains is taxable and commissions and fees can be collected. This can affect the choice of the investor with regard to stocks and portfolios.

Information is freely available to everyone; hence all investors have the same information and homogeneous expectations about the distribution of returns. Assumes that returns are distributed normally and specified by mean and standard deviation, the latter being a measure of risk. Usually that is not the case and returns might follow different distribution, which in turn refutes the assumption that the standard deviation is the appropriate measure of risk.

2.3 CAPM Implications

In equilibrium the demand of assets equals supply and the market portfolio is the mean variance efficient tangency portfolio. The market portfolio is a value-weighted portfolio, where the weight of each asset is the market value of the asset divided by the total market value of all assets.

The portfolio frontier is derived by combining the risk-free asset and the market portfolio. All investors hold identical risky portfolio, which is the tangent portfolio. Investors invest in risk free asset and a portfolio of risky assets. Risk averse investors hold more of the risk free asset, while risk tolerant investors give more weight to the risky asset.

The risk of an asset is determined by its covariability with the market portfolio. CAPM separates risk in systematic and non systematic. The systematic risk is correlated with the market portfolio and cannot be diversified and investors do not receive reward for it. The non systematic risk can be diversified and the higher the risk, the higher the return for taking it.

2.4 Single factor CAPM

2.4.1 Overview of the single factor CAPM

In 1959 Harry Markowitz developed the modern portfolio theory (MPT). According to the MPT stocks are related to each other and portfolio's return is a weighted combination of the returns of the assets it is comprised of. Markowitz discovered a positive relation between risk and return and showed how rational investors can use diversification to decrease risk.

A few years later William Sharpe (1964) and John Lintner (1965) used the MPT to create the well known Capital Asset Pricing Model (CAPM) that explains the relationship between the risk and the expected return and is used as a benchmark rate of return to evaluate an investment or to predict a price of a security. CAPM divides risk into systematic and nonsystematic risk. According to CAPM the risk premium of an asset/portfolio is proportional to its systematic risk measured by beta. Non-systematic risk can be completely eliminated through diversification and investors are only rewarded for carrying systematic risk.

2.4.2 The logic of the single factor CAPM

The CAPM uses the risk-free rate, the expected return of the market portfolio, and the beta of the asset to determine the expected return of an asset. Mathematically it is expressed as follows:

$$r_{it} - r_f = \alpha_i + \beta_{i,M}(r_{Mt} - r_f) + \varepsilon_{it} \quad (2.1)$$

where $\beta_{i,M}(r_M - r_f)$ is the market related part

and $\alpha_i + \varepsilon_i$ is the firm specific part

r_{it} return on an asset or portfolio i at time t

r_{Mt} return on the market as a whole during period t

r_f risk free rate of return

$\beta_{i,M}$ measure of the sensitivity of the asset's return on the changes in the market return

α_i a constant measuring an excess return

ε_{it} a return specific to the security for period t

If the expectations or the returns are taken instead, the model will look as follows:

$$E(r_{it}) - r_f = \alpha_i + \beta_i * [E(r_{Mt}) - r_f] + \varepsilon_{it} \quad (2.2)$$

$$E(r_{it}) - r_f = \alpha_i + [Cov(R_{it}, R_{Mt}) / Var(R_{Mt})] * [E(r_{Mt}) - r_f] + \varepsilon_{it} \quad (2.3)$$

Provided $E(r_{it}) - r_f > 0$, there is a linear relationship between the expected return on an asset and its beta, meaning the higher the beta on an asset, the higher the expected return and vice-versa.

- Alpha: The alpha is a measure of the excess return on an investment. CAPM states that investors are rewarded for the nonsystematic risk and the higher the risk of an asset, the higher the return. In reality a stock or portfolio may perform better or worse than expected and alpha measures the excess return of an asset over the risk adjusted reward.
- Beta: Beta is a measurement of the part of the volatility that represents systematic risk for an asset or a portfolio. It measures the sensitivity of asset's return to changes in the returns of the overall market. Beta also measures the contribution of an asset to the variance of the market portfolio, but not the volatility of the asset on its own. It also gives the amount of compensation the equity investors will receive for taking on additional risk.
- Sigma: The sigma of an asset measures its non-systematic risk, firm specific risk that is independent of the market.

The risk of the asset is measured by volatility in terms of the standard deviation:

$$Var(R_i) = \beta_i^2 \sigma_M^2 + \sigma^2(e_i), \quad (2.4)$$

where $\beta_i^2 \sigma_M^2$ is the market related part and $\sigma^2(e_i)$ is the firm specific part.

Investors are not compensated for the non systematic risk, the firm specific risk, as it is uncorrelated with the market as a whole and has no impact on a well diversified portfolio. The non systematic risk is viewed as random “noise” in the asset’s return, with zero mean and standard deviation that decreases with the addition of more assets. The expected return of the random noise is zero and it can be diversified away by adding more securities to the portfolio. By adding enough assets in a portfolio, the portfolio volatility approaches the volatility of the overall market. Thus, investors are rewarded only for the risk that cannot be diversified away, the systematic risk or market risk. Through diversification the volatility of an asset or portfolio of assets can be reduced without reducing expected returns. This can be achieved as long as asset returns are not perfectly correlated with each other. Adding more assets to a portfolio reduces volatility with a decreasing rate.

2.5 Fama and French three factor CAPM

2.5.1 Overview of the Fama and French three factor CAPM

In 1992 Fama and French introduced an alternative asset pricing model, an extension of CAPM that explains some of its anomalies. By adding two additional factors to the model - the firm size and book-to-market ratio, the Fama French three factor model captures much of the variation in returns caused by size effect, value effect, and other anomalies and improves the predictive power of CAPM.

Fama and French criticise CAPM for underestimating the expected return of companies with low beta and overestimating the expected returns of those with high beta. According to the Fama-French model small capitalization companies and companies with a high book-to-market ratio should expect a return premium as the small companies are more sensitive to changes in business conditions and companies with high book-to-market ration are more vulnerable to financial downturns.

2.5.2 The logic of the Fama French three factor model

The Fama-French model is a multiple regression model that explains expected returns with regard to their relationship with market risk, size risk and value risk and is defined as follows:

$$r_{it} - r_f = \alpha_i + \beta_{i,M} (r_{Mt} - r_f) + \beta_{i,SMB_t} (SMB_t) + \beta_{i,HML_t} (HML_t) + \varepsilon_{it} \quad (2.5)$$

r_{it}	return on stock i at time t
r_f	return on the risk-free asset
SMB _t	return on the size factor at time t
HML _t	return on the book-to-market factor at time t
ε_{it}	mean-zero regression disturbance

If the expected returns are taken instead the model becomes:

$$E(r_{it}) - r_f = \alpha_i + \beta_{i,M} (E(r_{Mt}) - r_f) + \beta_{i,SMB_t} E(SMB_t) + \beta_{i,HML_t} E(HML_t) + \varepsilon_{it} \quad (2.6)$$

By adding factors the Fama French model allows investors to better specify their risk exposure to the market, size and value risk. The beta due to market risk is calculated in the same way as in CAPM but will differ in value as weights are given to the other factors.

The SMB_t factor measures the excess return or the “size premium” that investors receive due to size risk and represents the difference between the returns on portfolios of small and big stocks:

$$SMB_t = r_{small} - r_{big} \quad (2.7)$$

A positive SMB indicates that small cap stocks have achieved better results than large cap stocks in time t. Small companies are considered as more risky as they are less diversified and subject to more risks.

The HML_t factor captures the risk exposure to value risk and is calculated as the return of portfolio of stocks with high ratios of book value to market value less the return on a portfolio of stocks with low book-to-market ratios:

$$HML = \text{high B/M minus low B/M} = r_{value} - r_{growth} \quad (2.8)$$

3 Data

The aim of this section is to describe how the data that will be used in the tests of the single and three factor CAPM have been selected and to present the argumentations behind the decisions. All tests will be applied to the US data as both models have been proved to work for the US market and to be able to test the effect of the stock market crash of October 1987 on the CAPM it is important that both models hold.

3.1 Risk free rate

The risk free rate in the capital asset pricing models is usually represented by the most marketable of all money market instruments US T-bills, T-notes or T-bonds. If for the same model a representative of the risk free rate with different maturity is used, that will lead to different results. To test whether the predictive power of the single and three factor models have changed after October 1987, I will use the one-month Treasury bill rate as monthly data will be applied to the returns and the aim is the time horizon of the representative to match the time horizon of the returns. Ibbotson Associates data for the one-month Treasury bill is used and the data are taken from K. French web site.¹

¹ http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

3.2 Expected return of market portfolio

According to CAPM the market portfolio consists of all assets in all markets, but in reality many assets are not included and a proxy for the market portfolio is used instead. Again as in the case of the risk free representative, if different proxy is used in the same model, different estimate for the returns will be generated. For my tests the expected return on the market is represented by the value-weight return on all NYSE, AMEX, and NASDAQ stocks taken from the Center for Research in Securities Prices (CRSP), the weight on asset “i” being calculated as follows:

$$w_i = \frac{P_i s_i}{\sum_{j=1}^N P_j s_j} \quad (3.1)$$

where N represents the number of stocks trading on NYSE/AMEX/NASDAQ.

3.3 Time period

To test whether the noticed effect of Black Monday October 1987 on stock market volatility has also influenced the predictive power of the single factor CAPM and the Fama French three factor CAPM, 230 monthly data observations before October 1987, starting August 1968 and ending September 1987, and 230 monthly data observations after October 1987, starting November 1987 and ending December 2006, will be examined. CAPM does not specify the length of the period that should be used and as was mentioned before, each time period will lead to different results, using daily data will provide a result that will differ from the result obtained by using weekly and monthly and yearly data. To catch the business cycle a longer period is preferred and the more observations the better. But since beta is time varying, the use of a longer period will provide a biased estimate of beta. On the other hand, it is not appropriate to compensate for the less observation due to the shortening of the period by using daily return as this will lead to more noise in the data and hence to less efficient results. Hence the choice was set on monthly data with a length of 20 years or 230 observations, before and after October 1987.

3.4 Portfolios

The evaluation tests of the single and three factor model will be performed on the 12 US industry portfolios: Consumer Non-durables (NoDur), Consumer Durables (Durbl), Manufacturing (Manuf), Energy (Enrgy), Chemicals (Chems), Business Equipment (BusEq), Telecommunications (Telcm), Utilities (Utils), Shops, Healthcare (Hlth), Money and Other. The data is provided by the Kenneth French web site¹ and a more detailed description of the portfolios is presented in Table 21 found in the Appendix. The choice of the number as well as the type of the portfolios was casual and should not be kept into these limits. The main idea behind the choice was to select a number that is neither too large, nor too small and to include the main industry groups. The number of industry portfolios can be increased or decreased and the tests can be applied to size and book-to-market portfolios as well in order to test whether the results are not sample specific.

¹ http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

3.5 SMB factor

SMB (Small Minus Big) is the average return on the three small portfolios minus the average return on the three big portfolios. The data is provided from the Kenneth French web site¹:

$$\begin{aligned} \text{SMB} = & 1/3 (\text{Small Value} + \text{Small Neutral} + \text{Small Growth}) \\ & - 1/3 (\text{Big Value} + \text{Big Neutral} + \text{Big Growth}) \end{aligned} \quad (3.2)$$

3.6 HML factor

HML (High Minus Low) is the average return on the two value portfolios minus the average return on the two growth portfolios. The data is provided from the Kenneth French web site¹:

$$\text{HML} = 1/2 (\text{Small Value} + \text{Big Value}) - 1/2 (\text{Small Growth} + \text{Big Growth}) \quad (3.3)$$

4 Methodology

The aim of this section is to present and justify the econometric methods that will be used to evaluate the single and three factor model. First the transformation and tests of the expected return and the method of ordinary least squares regression are shortly described, then the Monte Carlo Simulation technique is presented in detail.

Key words: OLS regression, JB normality test, Monte Carlo Simulation, Random variables, Probability distributions, Importance sampling.

4.1 Expected returns

The analysis will begin with a descriptive statistics of the returns and a normality test in order to determine how well the distribution of the returns is approximated with a normal distribution, as the first step in the Monte Carlo Simulation technique is to ascribe a probability distribution to the returns and defining the returns by a probability distribution that does not correspond to the real one and sampling from it will give incorrect results.

CAPM assumes that dividends are included in the returns on the portfolios. Prices are assumed to follow a lognormal distribution, while returns are normally distributed thus the returns are calculated as follows:

$$r_i = \ln P_t - \ln P_{t-1} \quad (4.1)$$

P_t price of an asset/portfolio in the day t

P_{t-1} price of an asset/portfolio in the day t-1

Next the mean, the variance, the skewness and the kurtosis will be found. I will apply the Jarque-Bera normality test to the returns:

¹ http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

$$JB = T \left[\frac{m_3^2 / m_2^3}{6} - \frac{[(m_4 / m_2^2) - 3]^2}{24} \right] + T \left[\frac{3m_1^2}{2m_2} + \frac{m_1 m_3}{m_2^2} \right] \sim \chi^2(2) \quad (4.2)$$

where m_1, m_2, m_3 and m_4 are respectively the mean, the variance, the skewness and the kurtosis. The asymptotic chi-square distribution with two degrees of freedom will be used to test the null hypothesis ($H_0 =$ normality), which is rejected when the test statistic is significant.

4.2 Regression analysis

I will proceed with a regression analysis to estimate the strength of the modeled relationship between the dependent variable, portfolio return, and the explanatory variables. I will use ordinary least-squares regression to estimate the alphas and betas of the 12 industry portfolios as OLS regression gives the best linear unbiased estimators, with the smallest mean squared error and the smallest variances of the estimates of the parameters.

For the regression analysis I will use the risk premiums of the returns. The regression equation for the single factor model then becomes:

$$E(R_{it}) = \alpha_i + \beta_{i,M} E(R_{Mt}) \quad (4.3)$$

and for the Fama French model:

$$E(R_{it}) = \alpha_i + \beta_{i,M} E(R_{Mt}) + \beta_{i,SMB} E(SMB_t) + \beta_{i,HML} E(HML_t) \quad (4.4)$$

I will test the following hypotheses for both models and will use a standardized test statistic to determine whether the hypothesis is correct or incorrect.

- Firstly I will test: $H_0: \alpha = 0$ versus $H_1: \alpha \neq 0$. According to CAPM everyone holds the market portfolio and each asset/portfolio generates an alpha of zero. If alpha has t-value greater that exceeds the 95% confidence interval there is evidence against H_0 and hence our model does not hold true.
- Secondly, I will test: $H_0: \beta = 0$ versus $H_1: \beta \neq 0$. If the regression slope coefficient is zero, this means that changes in the independent variable do not explain changes in the dependent variable. If the t-values for the betas exceeds the 95% confidence interval, betas are significant at a level of 5% and the H_0 that beta equals zero can be rejected.

I will use the coefficient of determination to estimate how well the estimated regression equation fitted the data. R^2 measures the proportion of an asset's total risk that is market risk

$$R^2 = \frac{\beta_i^2 \text{Var}(r_{Mt})}{\text{Var}(r_{it})} \quad (4.5)$$

For a multiple regression the R^2 is adjusted so as to take account of the number of variables added to the model.

$$\frac{R^2}{(1-R^2)} \frac{(n-k)}{k-1} \sim F_{k-1, n-k} \quad (4.6)$$

where n is number of observations and k is the number of independent regressors.

4.3 Monte Carlo Simulation

4.3.1 Definition

Monte Carlo Simulation is a technique that converts uncertainties in input variables of a model into probability distributions. By combining the distributions and randomly selecting values from them, it recalculates the simulated model many times and brings out the probability of the output.

4.3.2 Basic characteristics

Monte Carlo Simulation allows several inputs to be used at the same time to create probability distribution of one or more outputs.

The probability distributions assigned to the inputs of the model can be of different type. When the distribution is unknown, the one that represents the best fit could be chosen.

The use of random numbers characterizes Monte Carlo Simulation as a stochastic method. The random numbers have to be independent; no correlation should exist between them.

Monte Carlo Simulation generates the output as a range instead of a fixed value and shows how likely the output value is to occur in the range.

4.3.3 Random Variables

The use of random variables characterizes Monte Carlo Simulation as a stochastic method. Random variables are variables that behave in an uncertain way and a probability can be assigned to the possible values of the random variables. There are two types of random variables corresponding to the two types of distribution – discrete (probability distribution of variables that have certain discrete values) and continuous (probability distribution of variables that have values within infinite range). The discrete random variables take a specific number of real values and are defined by probability frequency function. The probabilities take values from zero to one and they sum to one:

$$\sum_{-\infty}^{\infty} P(x) = 1 \quad (4.7)$$

The continuous random variables take values between ∞ and $-\infty$, and are defined by probability density function. The probability of the value is defined as a probability that the random value \tilde{X} is less or equal to a specific value x:

$$F(x) = P(\tilde{X} \leq x) \text{ for } -\infty < x < \infty \quad (4.8)$$

The density function is obtain by taking the derivation of the distribution function:

$$f(x) = \frac{dF(x)}{dx} \quad (4.9)$$

The continuous random variable is defined by integrating the probability density function as follows:

$$\int_{-\infty}^{\infty} f(x)dx = 1 \quad (4.10)$$

Random variables can be described by their probability distributions and their moments: mean, variance, skewness and kurtosis.

4.3.4 Probability Distributions

One of the basic characteristics of the Monte Carlo Simulation technique is that it works with probability distributions. The MCS simulation process starts with randomly selecting values from probability distributions of the input variables of a model, thus combining the distributions and generating the probability of the output.

As mentioned earlier, prices are assumed to follow a lognormal distribution, while returns are normally distributed. The reasoning behind the assumption is that allowing for negative values, the normal distribution is not appropriate to be used as prices cannot become negative. Thus for the Monte Carlo Simulation of single and three factor model, Normal/Gaussian distribution will be assigned to the returns.

Normal/Gaussian Distribution is a continuous distribution completely defined by its first two moments - the mean and the standard deviation. The mean is also the mode and the median.

$$f(x; \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2} \quad (4.11)$$

where, μ is the mean and σ is the standard deviation of the distribution.

The normal probability distribution is symmetric around the expected value, which is equal to the mean of the distribution, meaning that positive and negative deviations from the mean are equally likely to occur. And the larger the deviation the less likely is to occur.

In the Monte Carlo Simulation of the models the following property will be used: if normally distributed random variables are weighted and summed than the result will be also a random variable with normal distribution. The addition of a constant or multiplication with a constant will not change the distribution type.

4.3.5 The Mathematics behind Monte Carlo Simulation

Consider that we have a real-valued function $g(X)$, with probability frequency function $P(x)$, if X is discrete, or probability density function $f(x)$, if X is continuous. Then we can define the expected value of $g(X)$ in discrete and continuous terms respectively:

$$E(g(X)) = \sum_{-\infty}^{\infty} g(x)P(x), \text{ where } P(x) > 0 \text{ and } \sum_{-\infty}^{\infty} P(x) = 1 \quad (4.12)$$

$$\text{or } E(g(X)) = \int_{-\infty}^{\infty} g(x)f(x)dx, \text{ where } f(x) > 0 \text{ and } \int_{-\infty}^{\infty} f(x)dx = 1 \quad (4.13)$$

Next make n random drawings of X (x_1, \dots, x_n), called trial runs or simulation runs, calculate $g(x_1), \dots, g(x_n)$ and find the mean of $g(x)$ of the sample:

$$\hat{g}_n(x) = \frac{1}{n} \sum_{i=1}^n g(x_i) \quad (4.14)$$

this equation represents the final simulated value of $E(g(X))$. Therefore

$$\hat{g}_n(X) = \frac{1}{n} \sum_{i=1}^n g(X_i) \quad (4.15)$$

will be the Monte Carlo estimator of $E(g(X))$.

“The Law of Large Numbers says that in repeated, independent trials with the same probability p of success in each trial, the chance that the percentage of successes differs from the probability p by more than a fixed positive amount, $e > 0$, converges to zero as the number of trials n goes to infinity, for every positive e .”(P.B.Stark)¹

Or if $X_1 + \dots + X_n$ are identically and independently distributed with a mean μ , then for any $e > 0$:

$$\lim_{n \rightarrow \infty} P\left(\left|\frac{X_1 + \dots + X_n}{n} - \mu\right| > e\right) \rightarrow 0 \text{ as } n \rightarrow \infty \quad (4.16)$$

Thus as $n \rightarrow \infty$, $\lim_{n \rightarrow \infty} P\left(\left|\hat{g}_n(X) - E(g(X))\right| > e\right) \rightarrow 0$ and $\hat{g}_n(X) \rightarrow E(g(X))$.

Hence $\hat{g}_n(X)$ is unbiased estimate of $E(g(X))$ and

$$E(\hat{g}_n(X)) = E\left(\frac{1}{n} \sum_{i=1}^n g(X_i)\right) = \frac{1}{n} \sum_{i=1}^n E(g(X_i)) = E(g(X)) \quad ** \quad (4.17)$$

We are now able to find the variance of $\hat{g}_n(X)$:

$$Var(\hat{g}_n(X)) = Var\left(\frac{1}{n} \sum_{i=1}^n g(X_i)\right) = \frac{Var(g(X))}{n} = \frac{1}{n} \int_{x \in \mathcal{X}} [g(x) - E(g(X))]^2 f_{\mathcal{X}}(x) dx \quad ** \quad (4.18)$$

Next the dispersion around the estimated mean with the unbiased variance of $\hat{g}_n(X)$ can also be found:

$$\widehat{Var}(\hat{g}_n(X)) = \frac{1}{n} \frac{1}{n-1} \sum_{i=1}^n (g(x_i) - \hat{g}_n(x))^2 \quad ** \quad (4.19)$$

Thus all of the information is now available to generate the additional descriptive statistics: minimum and maximum values, range width, skewness, kurtosis, standard errors and percentiles.

4.3.6 Importance Sampling

Monte Carlo Simulation assigns probability distributions to the inputs of the model but defining the uncertainty of an input value by a probability distribution that does not correspond to the real one and sampling from it might produce incorrect result that is why the choice of a good distribution from which to sample random variables is very important. The importance sampling technique aims to concentrate the distribution of the sample points to a region that is of greatest importance instead of spreading them evenly. Consider n

¹ <http://stat-www.berkeley.edu/~stark/Java/Html/lln.htm>

** Anderson C. Eric, “Monte Carlo Methods and Importance Sampling”, Lecture Notes

dimensional integral $\int g(x)dx$, the function $f(x)$ might not be the best probability density function to be used hence a new function $\tilde{f}(x)$ is introduced:

$$\int g(x)dx = \int g(x) \frac{\tilde{f}(x)}{f(x)} dx = \int \frac{g(x)}{\tilde{f}(x)} \tilde{f}(x) dx = E_{\tilde{h}} \left(\frac{g(x)}{\tilde{f}(x)} \right), \quad (4.20)$$

where $\tilde{f}(x) > 0$ and $\int \tilde{f}(x)dx = 1$.

A good importance sampling function $\tilde{f}(x)$ should be close to being proportional to $|g(x)|$, it should be easy to estimate values from $\tilde{f}(x)$ and to compute the density $\tilde{f}(x)$ for any value x that you might be realized.

4.3.7 Applying MCS to the single and three factor CAPM

First I define $g(X)$ for the single and three factor capital asset pricing models:

For the single factor model:

$$g(x) = E(R_i) = \alpha_i + \beta_{i,M} E(R_M) \quad (4.21)$$

For the three factor model:

$$g(x) = E(R_i) = \alpha_i + \beta_{i,M} E(R_M) + \beta_{i,SMB} E(SMB) + \beta_{i,HML} E(HML) \quad (4.22)$$

As explained earlier the returns for the both models will be specified by a Normal/Gaussian distribution. The coefficients alpha, beta, SMB, HML generated by the OLS regression will be used as assumptions for the MCS. The lower and upper bound for 95% confidence interval for each of the coefficients taken from the OLS regression will be set as limits for the coefficients.

For the Monte Carlo Simulation of the single factor model and the three factor model I will perform 10000 simulation runs ($n=10000$) because the number of trials in the MCS should not be too small, as it might not be sufficient to simulate the model and clustering of values may occur.

Then I will calculate the Monte Carlo simulated value of $E(g(X))$ for both models as follows:

$$\hat{g}_n(x) = \frac{1}{10000} \sum_{j=1}^{10000} g(x_j) = \widehat{E(R_i)} = \frac{1}{10000} \sum_{j=1}^{10000} [\alpha_j + \beta_j E(R_{Mj})] \quad (4.23)$$

for the single factor model, and

$$\begin{aligned}\widehat{g}_n(x) &= \frac{1}{10000} \sum_{j=1}^{10000} g(x_j) = \widehat{E(R_i)} = \\ &= \frac{1}{10000} \sum_{j=1}^{10000} [\alpha_j + \beta_j E(R_{M_j}) + \beta_{SMB_j} E(SMB_j) + \beta_{HML_j} E(HML_j)]\end{aligned}\quad (4.24)$$

for the three factor model, where $j=1, \dots, 10\,000$ signifies the number of the variables of the corresponding Monte Carlo Simulation run.

Thus I can also find the dispersion around the mean for the single factor model:

$$\widehat{Var}(\widehat{g}_n(X)) = \frac{1}{10000} \frac{1}{10000-1} \sum_{j=1}^{10000} \left([\alpha_j + \beta_j E(R_{M_j})] - \left[\frac{1}{10000} \sum_{j=1}^{10000} [\alpha_j + \beta_j E(R_{M_j})] \right] \right)^2 \quad (4.25)$$

and for the three factor model:

$$\begin{aligned}\widehat{Var}(\widehat{g}_n(X)) &= \frac{1}{10000} \frac{1}{10000-1} \sum_{j=1}^{10000} \left([\alpha_j + \beta_j E(R_{M_j}) + \beta_{SMB_j} E(SMB_j) + \beta_{HML_j} E(HML_j)] - \right. \\ &\quad \left. - \left[\frac{1}{10000} \sum_{j=1}^{10000} [\alpha_j + \beta_j E(R_{M_j}) + \beta_{SMB_j} E(SMB_j) + \beta_{HML_j} E(HML_j)] \right] \right)^2\end{aligned}\quad (4.26)$$

Knowing the Monte Carlo estimators of the mean and variance I will next calculate the skewness and kurtosis as well as the standard error of the simulated returns.

5 Empirical results and analysis

The regression results on the 12 industry portfolios for the single factor model and the Fama French model for the periods before and after October 1987 are presented in the tables 17, 18, 19 and 20 found in the Appendix. The t-statistic of all alphas of the single-factor model for the period 1968 – 1987 and for the period 1987 – 2006 show that alphas are not significant, hence the single-factor model holds. The alphas of the Fama French model are also insignificant with one exception – portfolio Durables for the period after October 1987. The regression results also show that all of the betas for the both models at the different time periods are significant at a level of 5%.

Single Factor CAPM 1968 - 1987			Single Factor CAPM 1987 - 2006		
Alphas	min	-0.201	Alphas	min	-0.314
	max	0.324		max	0.487
	MAV	0.141		MAV	0.273
Beta	min	0.626	Beta	min	0.325
	max	1.202		max	1.598
Rsqr	min	0.489	Rsqr	min	0.116
	max	0.919		max	0.817
Fama French CAPM 1968 - 1987			Fama French CAPM 1987 - 2006		
Alphas	min	-0.194	Alphas	min	-0.717
	max	0.535		max	0.481
	MAV	0.148		MAV	0.182
Beta	min	0.735	Beta	min	0.615
	max	1.082		max	1.307
Rsqr	min	0.537	Rsqr	min	0.353
	max	0.948		max	0.834

Table 1 Coefficients minimum, maximum, MAV values

Looking at Table 1, it is clear that the mean absolute values (MAV) of the alphas before October 1987 are lower for the both models (0,141 for the single factor model and 0,148 for the Fama French model) than the MAV of the alphas after October 1987 for the both models (0,273 for the single factor model and 0,182 for the Fama French model). This tells us that the performance of the both models has worsened. Furthermore, by looking at the MAV of the single factor model and Fama French model we can also see that the MAV of the both models before October 1987 are closer to each other, difference of only 0,007, than are the MAV of the both models after October 1987, where the difference in MAV increases to 0,091. All of the said above indicates that the Black Monday of October 1987 has influenced the predictive power of the single factor model and Fama French model.

With regard to the R-squared, it is interesting to notice that the ranges of the goodness of fit measures of the both models before October 1987 are higher ([0.489, 0.919] for the single factor model and [0.735, 1.082] for the Fama French model) compared to the ranges of the goodness of fit measures after October 1987 ([0.116, 0.817] for the single factor model and [0.615, 1.307] for the Fama French model). Looking at Table 2 below it is evident that the single factor CAPM before October 1987 has much higher R-squared for all industry portfolios compared to the R-squared for the period after October 1987, the same applies for the Fama French model R-squared results with two exceptions – Business equity and Telecom portfolios. As two more factors play role in the Fama French model, it is understandable why there are exceptions and why the R-squared values are not so distinctively different before and after October 1987 as are the R-squared values of the single factor CAPM. Since R-squared estimates how well the estimated regression equation fits the data, we can conclude that before October 1987 both models have greater explanatory power when compared to the period after October 1987.

Industry portfolios	Single Factor CAPM 1968 - 1987			Single Factor CAPM 1987 - 2006			Fama French CAPM 1968 - 1987				Fama French CAPM 1987 - 2006					
	Coefficients		Rsqr	Coefficients		Rsqr	Coefficients				Rsqr	Coefficients				Rsqr
	Alpha	Beta		Alpha	Beta		Alpha	Beta	SMB	HML		Alpha	Beta	SMB	HML	
NoDur	0.264	0.937	0.831	0.334	0.637	0.437	0.213	0.922	0.128	0.064	0.837	0.164	0.777	-0.185	0.275	0.527
Durbl	0.074	1.003	0.727	-0.180	1.013	0.509	-0.080	1.032	0.104	0.239	0.742	-0.717	1.307	0.184	0.815	0.643
Manuf	-0.054	1.083	0.919	0.186	0.991	0.778	-0.067	1.063	0.093	0.005	0.922	-0.023	1.102	0.091	0.315	0.810
Enrgy	0.224	0.925	0.533	0.487	0.578	0.240	0.209	1.051	-0.451	0.099	0.576	0.117	0.807	-0.008	0.572	0.353
Chems	0.084	0.980	0.854	0.209	0.720	0.488	0.099	1.012	-0.139	-0.001	0.860	0.008	0.872	-0.147	0.321	0.570
BusEq	-0.201	1.099	0.773	-0.314	1.598	0.727	0.009	0.979	0.158	-0.374	0.804	0.188	1.258	0.172	-0.786	0.834
Telcm	0.324	0.626	0.489	-0.147	0.987	0.588	0.142	0.735	-0.154	0.328	0.537	-0.080	0.997	-0.267	-0.085	0.616
Utils	0.070	0.689	0.555	0.417	0.325	0.116	-0.194	0.834	-0.178	0.468	0.640	-0.026	0.615	-0.090	0.690	0.401
Shops	0.103	1.110	0.787	0.118	0.941	0.650	0.073	1.040	0.301	0.000	0.807	0.007	1.008	0.006	0.171	0.660
Hlth	0.152	0.929	0.643	0.305	0.717	0.417	0.535	0.853	-0.245	-0.595	0.740	0.481	0.676	-0.351	-0.248	0.481
Money	0.062	1.056	0.846	0.352	0.944	0.637	-0.064	1.081	0.083	0.197	0.857	0.003	1.193	-0.179	0.551	0.789
Other	-0.073	1.202	0.901	-0.225	1.007	0.817	-0.093	1.082	0.477	-0.045	0.948	-0.366	1.070	0.126	0.210	0.834

Table 2 Summarized regression results for Single Factor CAPM and Fama French CAPM for the time periods before and after October 1987

The Monte Carlo simulation results on the 12 industry portfolios for the single factor model and the Fama French model for the periods before and after October 1987 are presented in Table 3-14. As it can be seen from Table 3, when a Monte Carlo simulation technique is applied to the returns of the Consumer Non-durables portfolio, the simulation returns for Fama French model for both time periods are closer to the actual ones compared to the simulation returns for single factor model, which confirms the superiority of the former capital pricing model. Furthermore, the simulated returns before October 1987 for both models are also closer to the actual ones than are the simulated returns after October 1987 or the simulation imply that before October 1987 the returns could be better predicted. The same is the case with the Monte Carlo Simulation of the standard deviation. The simulation results support the previous findings of superiority of the Fama French model over the single factor model as the simulated standard deviation for the Fama French model are closer to the actual one. The simulated standard deviations of both models before October 1987 are better approximations of the actual standard deviations (difference of 0,45 for the single factor model and 0,55 for the three factor model before October 1987 versus difference of 1,35 for the single factor model and 0,59 for the three factor model after October 1987), meaning that the models have measured the uncertainty related to the expected returns before the Black Monday better than after the stock market crash.

Consistent with the findings above and a clear evidence in support of the hypothesis that the Black Monday has influenced the predictability of the single factor model is the fact that all of the simulated standard deviations of the returns of the portfolios Consumer Durables, Manufacturing, Energy, Chemicals, Business Equity, Utilities, Shops, Health, Money, and Other before October 1987 better approximate the corresponding real standard deviations for the same period than the simulated standard deviations of those portfolios after October 1987 approximate the corresponding real standard deviations for the same time period. From Table 4, it can be seen that for the Consumer Durables portfolio the difference between the standard deviations generated by the OLS regression and the MCS is 0,83 before October 1987 versus 1,68 after October 1987. For the Manufacturing portfolio (Table 5) the OLS regression and MCS estimations of the standard deviation are closer to each other and again before October 1987 the difference is smaller than after October 1987: 0,20 versus 0,51. Although the differences between the OLS regression and MCS estimations of the standard deviation for the Energy portfolio (Table 6) are larger, they still are consistent with the findings above, namely before October 1987 the difference is smaller than after October 1987: 1,64 before

October 1987 versus 2,45 after October 1987. Similarly, for the Chemicals portfolio (Table 7) the simulated standard deviation better approximates the standard deviation generated by the OLS regression before October 1987, a difference between the generated standard deviations by OLS regression and MCS of 0,35 before October 1987 versus 1,26 after October 1987. When Monte Carlo simulation technique is applied to the returns of the Business Equity portfolio (Table 8) it again approximates the OLS regression standard deviation better before October 1987, namely a difference of 0,74 before October 1987 versus 1,05 after October 1987. Analogous to the results obtained for the Energy portfolio are those for the Utilities portfolio (Table 10), namely larger differences between OLS regression and MSC estimations of the standard deviations but smaller before October 1987: 1,11 before October 1987 versus 2,56 after October 1987. The OLS regression and MCS estimations of the standard deviation for Shops portfolio (Table 11) are relatively close and the difference between them is smaller before October 1987: 0,70 versus 0,93 after October 1987. Looking at the Health portfolio (Table 12) it can be seen that it gives higher differences between the OLS regression and MCS estimations of the standard deviation but again the difference is smaller before October 1987: 1,09 before October 1987 versus 1,60 after October 1987. For the Money portfolio (Table 13) and Other portfolio (Table 14) the differences between the OLS regression and MCS estimations of the standard deviation are smaller before October 1987 than after October 1987, correspondingly 0,38 before October 1987 versus 0,93 after October 1987 and 0,32 before October 1987 versus 0,45 after October 1987. As standard deviation measures the dispersion of the returns, the single factor model simulation results for all of the portfolios above confirm that the returns are not so widely spread as predicted by the single factor model and that before October 1987 the single factor CAPM can better capture the uncertainty in the difference of the expected and actually earned returns of the portfolios. There is only one portfolio, the Telecom portfolio, that is not consistent with the findings but if we refer back to Table 2, we can see that for the Telecom portfolio both models were giving contradictive results for the constant before and after October 1987, meaning that there might be other factors related to this portfolio that have explanatory power and that the single and three factor model do not account for.

For the Fama French model the results are to a higher extent similar. The simulated standard deviations of the Energy, Chemicals, Business Equity, Telecom, Utilities, Health and Money portfolios for the period before October 1987 better approximate the real standard deviations and suggest that the model is more efficient before the stock market crash in 1987. For the Energy portfolio (Table 6) the difference between the simulated and the OLS regression standard deviation before October 1987 is 0,83 versus 1,00 after October 1987. For the Chemicals portfolio (Table 7) the difference between the simulated standard deviations and the standard deviation generated by the OLS regression is 0,26 before October 1987 versus 0,43 after October 1987 and again the difference is smaller before October 1987. For the Business Equity portfolio (Table 8) the differences between the OLS regression and MCS estimations of the standard deviation are larger but in agreement with the findings above, namely before October 1987 the difference is smaller than after October 1987: 1,14 before October 1987 versus 1,90 after October 1987. When Monte Carlo simulation technique is applied to the returns of Telecom portfolio (Table 9) it approximates the OLS regression standard deviation better before October 1987, namely a difference of 0,63 before October 1987 versus 1,10 after October 1987. The OLS regression and MCS estimations of the

standard deviation for Utilities portfolio (Table 10) are closer to each other and before October 1987 the difference is smaller than after October 1987: 0,20 before October 1987 versus 0,51 after October 1987. For the Health portfolio (Table 12) and Money portfolio (Table 13) the OLS regression and MCS estimations of the standard deviation are also smaller before October 1987, correspondingly 1,05 before October 1987 versus 1,41 after October 1987 and 0,27 before October 1987 versus difference of 0,33 after October 1987. Still, there are four portfolios, Durables, Manufacturing, Shops and Other portfolios, that are not consistent with the findings. As mentioned earlier in the analysis, a reason for not so distinctive evidence in support of the hypothesis that the Fama French model performs better for the period before October 1987 is the inclusion of the SMB and HML factor that explain some of the deviations that the market beta cannot account for and thus make the Fama French model superior to the single factor CAPM and moderate the effect of market beta on the portfolio returns.

6 Conclusion

In this thesis I have performed tests to examine whether the noticed effect of Black Monday October 1987 on volatility has also influenced the predictive power of the single factor CAPM and the Fama French three factor CAPM, in order to conclude whether the models are less effective after the stock market crash.

First I have applied OLS regression on the 12 Industry portfolios for the single factor model and the Fama French model for the periods before and after October 1987. The regression results concluded that all of the betas are significant at a level of 5% for both models and for both time periods that are examined. The t-statistic of all alphas of the 12 portfolios before and after October 1987 showed that alphas are not significant and that both models hold with one exception, Durables portfolio for the Fama French model for the period after October 1987. Furthermore, the mean absolute values of the alphas before October 1987 were lower compared to the mean absolute values of the alphas after October 1987, implying that the performance of the both models has worsened. An important result of the OLS regression in support of the hypothesis that the Black Monday event has influenced the predictive power of the single factor model is that the single factor model has much higher R-squared for all industry portfolios hence greater explanatory power of the model before October 1987. The same R-squared results were found for the Fama French model with two exceptions – Business equity and Telecom portfolios. The not so distinctive difference in the R-squared values before and after October 1987 for the Fama French model is due to the inclusion of the two new explanatory variables the SMB and HML that moderate the effect of the market beta on the returns in the three factor model.

In addition, when Monte Carlo Simulation technique was applied to both models for periods before and after October 1987 for the 12 industry portfolios, the results again gave stronger evidence in support of the hypothesis that the single factor model has performed better before October 1987, namely the results of eleven out of twelve simulated industry portfolios, Consumer Non-durables, Consumer Durables, Manufacturing, Energy, Chemicals, Business

Equity, Utilities, Shops, Health, Money, and Other, have confirmed the means of the returns after October 1987 are too far from the predicted or that before October 1987 the single factor CAPM can better capture the uncertainty in the difference of the expected and actually earned returns of the eleven portfolios. For the Fama French model eight out of twelve industry portfolios, Consumer Non-durables, Energy, Chemicals, Business Equity, Telecom, Utilities, Health and Money, are consistent with the findings in support of the hypothesis for the worsened efficiency of the model after the stock market crash in October 1987.

The Monte Carlo Simulation technique is straightforward and flexible. It converts the uncertainties in the input variables of a model into probability distributions and allows several inputs to be used at the same time. By combining the distributions and randomly selecting values from them, MSC recalculates the simulated model many times and brings out the probability of the output. Monte Carlo Simulation generates the output as a range instead of a fixed value and shows how likely the output value is to occur in the range. All these make the Monte Carlo Simulation technique an appropriate and preferred for evaluation of the accuracy and performance of other models but it has a few drawbacks. In the first place, the assumption that the input variables are independent might not be valid and misleading results might come from inputs that are mutually exclusive or in case significant correlation is found between two or more input distributions. Secondly, Monte Carlo Simulation cannot wipe out uncertainty. It only helps to better understand the uncertainty and how it affects the forecasted variables by ascribing probabilistic characteristics to the inputs and outputs of a model.

Suggestions for further research

In addition to the study I have presented in the master thesis, it would be relevant and interesting to shorten the time period and repeat the tests. This will give better insight on whether the changes in volatility observed after the stock market crash in October 1987 have changed the efficiency of the single factor CAPM and the Fama French CAPM and whether there is short term or long term effect on the predictive power of the models. Furthermore, the number of industry portfolios can be switched as well and the tests can be applied to size and book-to-market portfolios as well in order to test whether the results are not sample specific.

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Table 3 Monte Carlo Simulation results for Consumer Non Durables portfolio for Single factor CAPM and Fama French CAPM before and after October 1987. (*indicates Monte Carlo Simulation results)

Monte Carlo Simulation results Single Factor CAPM before and after October 1987							
Portfolio	Model	Mean		Std. Deviation	Variance	Skewness	Kurtosis
		Statistic	Std. Error	Statistic	Statistic	Statistic	Statistic
NoDur 87/06	SF	0.757	0.258	3.916	15.335	-0.158	1.021
NoDur 87/06*	SF	0.830	0.026	2.565	6.581	-0.042	3.036
	SF	-0.073	0.233	1.351	8.754	-0.116	-2.015
NoDur 68/87	SF	0.591	0.320	4.852	23.544	0.023	1.166
NoDur 68/87*	SF	0.652	0.044	4.406	19.414	0.012	3.011
	SF	-0.061	0.276	0.446	4.130	0.011	-1.845

Monte Carlo Simulation results Fama French CAPM before and after October 1987							
Portfolio	Model	Mean		Std. Deviation	Variance	Skewness	Kurtosis
		Statistic	Std. Error	Statistic	Statistic	Statistic	Statistic
NoDur 87/06	FF	0.757	0.258	3.916	15.335	-0.158	1.021
NoDur 87/06*	FF	0.779	0.033	3.325	11.053	0.016	3.081
	FF	-0.022	0.225	0.591	4.282	-0.173	-2.060
NoDur 68/87	FF	0.591	0.320	4.852	23.544	0.023	1.166
NoDur 68/87*	FF	0.611	0.043	4.306	18.544	-0.013	3.071
	FF	-0.020	0.277	0.546	5.000	0.036	-1.905

Table 4 Monte Carlo Simulation results for Consumer Durables portfolio for Single factor CAPM and Fama French CAPM before and after October 1987. (*indicates Monte Carlo Simulation results)

Monte Carlo Simulation results Single Factor CAPM before and after October 1987							
Portfolio	Model	Mean		Std. Deviation	Variance	Skewness	Kurtosis
		Statistic	Std. Error	Statistic	Statistic	Statistic	Statistic
Durbl 87/06	SF	0.492	0.380	5.767	33.254	-0.286	0.718
Durbl 87/06*	SF	0.515	0.041	4.089	16.719	-0.025	3.051
	SF	-0.023	0.339	1.678	16.535	-0.261	-2.334
Durbl 68/87	SF	0.424	0.366	5.550	30.799	0.333	0.921
Durbl 68/87*	SF	0.364	0.047	4.720	22.283	0.012	3.068
	SF	0.060	0.319	0.829	8.517	0.322	-2.147

Monte Carlo Simulation results Fama French CAPM before and after October 1987							
Portfolio	Model	Mean		Std. Deviation	Variance	Skewness	Kurtosis
		Statistic	Std. Error	Statistic	Statistic	Statistic	Statistic
Durbl 87/06	FF	0.492	0.380	5.767	33.254	-0.286	0.718
Durbl 87/06*	FF	0.570	0.060	5.954	35.447	-0.007	3.028
	FF	-0.078	0.321	-0.187	-2.193	-0.279	-2.310
Durbl 68/87	FF	0.424	0.366	5.550	30.799	0.333	0.921
Durbl 68/87*	FF	0.403	0.050	4.953	24.534	0.031	3.000
	FF	0.021	0.316	0.596	6.265	0.302	-2.078

Table 5 Monte Carlo Simulation results for Manufacturing portfolio for Single factor CAPM and Fama French CAPM before and after October 1987. (*indicates Monte Carlo Simulation results)

Monte Carlo Simulation results Single Factor CAPM before and after October 1987							
Portfolio	Model	Mean		Std. Deviation	Variance	Skewness	Kurtosis
		Statistic	Std. Error	Statistic	Statistic	Statistic	Statistic
Manuf 87/06	SF	0.844	0.301	4.564	20.826	-0.493	0.987
Manuf 87/06*	SF	0.867	0.041	4.052	16.421	-0.038	3.043
	SF	-0.022	0.260	0.511	4.405	-0.456	-2.056
Manuf 68/87	SF	0.324	0.351	5.329	28.402	0.124	0.563
Manuf 68/87*	SF	0.323	0.051	5.133	26.348	0.049	2.999
	SF	0.001	0.300	0.196	2.053	0.076	-2.436
Monte Carlo Simulation results Fama French CAPM before and after October 1987							
Portfolio	Model	Mean		Std. Deviation	Variance	Skewness	Kurtosis
		Statistic	Std. Error	Statistic	Statistic	Statistic	Statistic
Manuf 87/06	FF	0.844	0.301	4.564	20.826	-0.493	0.987
Manuf 87/06*	FF	0.878	0.046	4.618	21.329	-0.017	3.000
	FF	-0.033	0.255	-0.055	-0.503	-0.477	-2.013
Manuf 68/87	FF	0.324	0.351	5.329	28.402	0.124	0.563
Manuf 68/87*	FF	0.294	0.050	4.952	24.522	0.011	3.018
	FF	0.030	0.302	0.377	3.880	0.114	-2.455

Table 6 Monte Carlo Simulation results for Energy portfolio for Single factor CAPM and Fama French CAPM before and after October 1987. (*indicates Monte Carlo Simulation results)

Monte Carlo Simulation results Single Factor CAPM before and after October 1987							
Portfolio	Model	Mean		Std. Deviation	Variance	Skewness	Kurtosis
		Statistic	Std. Error	Statistic	Statistic	Statistic	Statistic
Enrgy 87/06	SF	0.871	0.316	4.798	23.022	0.391	0.975
Enrgy 87/06*	SF	0.865	0.023	2.348	5.511	-0.016	3.016
	SF	0.007	0.293	2.450	17.510	0.407	-2.041
Enrgy 68/87	SF	0.547	0.395	5.984	35.805	0.208	1.179
Enrgy 68/87*	SF	0.579	0.043	4.342	18.850	0.040	3.022
	SF	-0.032	0.351	1.642	16.954	0.168	-1.843
Monte Carlo Simulation results Fama French CAPM before and after October 1987							
Portfolio	Model	Mean		Std. Deviation	Variance	Skewness	Kurtosis
		Statistic	Std. Error	Statistic	Statistic	Statistic	Statistic
Enrgy 87/06	FF	0.871	0.316	4.798	23.022	0.391	0.975
Enrgy 87/06*	FF	0.848	0.038	3.798	14.429	0.051	2.949
	FF	0.024	0.278	1.000	8.593	0.340	-1.974
Enrgy 68/87	FF	0.547	0.395	5.984	35.805	0.208	1.179
Enrgy 68/87*	FF	0.631	0.052	5.150	26.525	-0.023	3.034
	FF	-0.084	0.343	0.833	9.280	0.232	-1.855

Table 7 Monte Carlo Simulation results for Chemicals portfolio for Single factor CAPM and Fama French CAPM before and after October 1987. (*indicates Monte Carlo Simulation results)

Monte Carlo Simulation results Single Factor CAPM before and after October 1987							
Portfolio	Model	Mean		Std. Deviation	Variance	Skewness	Kurtosis
		Statistic	Std. Error	Statistic	Statistic	Statistic	Statistic
Chems 87/06	SF	0.688	0.276	4.188	17.541	-0.206	0.462
Chems 87/06*	SF	0.665	0.029	2.926	8.559	0.019	2.982
	SF	0.023	0.247	1.263	8.981	-0.226	-2.519
Chems 68/87	SF	0.427	0.330	5.004	25.037	0.299	1.205
Chems 68/87*	SF	0.358	0.047	4.655	21.672	-0.007	3.114
	SF	0.069	0.283	0.348	3.365	0.306	-1.909

Monte Carlo Simulation results Fama French CAPM before and after October 1987							
Portfolio	Model	Mean		Std. Deviation	Variance	Skewness	Kurtosis
		Statistic	Std. Error	Statistic	Statistic	Statistic	Statistic
Chems 87/06	FF	0.688	0.276	4.188	17.541	-0.206	0.462
Chems 87/06*	FF	0.728	0.038	3.761	14.143	0.042	3.077
	FF	-0.040	0.239	0.427	3.397	-0.249	-2.615
Chems 68/87	FF	0.427	0.330	5.004	25.037	0.299	1.205
Chems 68/87*	FF	0.386	0.047	4.741	22.475	-0.064	2.946
	FF	0.041	0.283	0.263	2.562	0.363	-1.740

Table 8 Monte Carlo Simulation results for Business Equipment portfolio for Single factor CAPM and Fama French CAPM before and after October 1987. (*indicates Monte Carlo Simulation results)

Monte Carlo Simulation results Single Factor CAPM before and after October 1987							
Portfolio	Model	Mean		Std. Deviation	Variance	Skewness	Kurtosis
		Statistic	Std. Error	Statistic	Statistic	Statistic	Statistic
BusEq 87/06	SF	0.748	0.502	7.615	57.990	-0.234	0.796
BusEq 87/06*	SF	0.760	0.066	6.569	43.157	-0.005	3.028
	SF	-0.012	0.436	1.046	14.833	-0.228	-2.233
BusEq 68/87	SF	0.183	0.389	5.898	34.785	0.263	0.535
BusEq 68/87*	SF	0.177	0.052	5.162	26.646	0.023	3.051
	SF	0.006	0.337	0.736	8.138	0.241	-2.516

Monte Carlo Simulation results Fama French CAPM before and after October 1987							
Portfolio	Model	Mean		Std. Deviation	Variance	Skewness	Kurtosis
		Statistic	Std. Error	Statistic	Statistic	Statistic	Statistic
BusEq 87/06	FF	0.748	0.502	7.615	57.990	-0.234	0.796
BusEq 87/06*	FF	0.763	0.057	5.716	32.668	0.007	3.027
	FF	-0.016	0.445	1.900	25.322	-0.240	-2.231
BusEq 68/87	FF	0.183	0.389	5.898	34.785	0.263	0.535
BusEq 68/87*	FF	0.169	0.048	4.755	22.609	0.021	2.951
	FF	0.014	0.341	1.143	12.175	0.243	-2.416

Table 9 Monte Carlo Simulation results for Telecom portfolio for Single factor CAPM and Fama French CAPM before and after October 1987. (*indicates Monte Carlo Simulation results)

Monte Carlo Simulation results Single Factor CAPM before and after October 1987							
Portfolio	Model	Mean		Std. Deviation	Variance	Skewness	Kurtosis
		Statistic	Std. Error	Statistic	Statistic	Statistic	Statistic
Telcm 87/06	SF	0.509	0.345	5.227	27.324	-0.198	1.517
Telcm 87/06*	SF	0.513	0.040	4.002	16.015	-0.010	3.039
	SF	-0.004	0.305	1.225	11.309	-0.188	-1.523
Telcm 68/87	SF	0.543	0.279	4.225	17.852	0.139	-0.158
Telcm 68/87*	SF	0.576	0.030	2.957	8.743	0.012	2.929
	SF	-0.033	0.249	1.268	9.109	0.128	-3.087
Monte Carlo Simulation results Fama French CAPM before and after October 1987							
Portfolio	Model	Mean		Std. Deviation	Variance	Skewness	Kurtosis
		Statistic	Std. Error	Statistic	Statistic	Statistic	Statistic
Telcm 87/06	FF	0.509	0.345	5.227	27.324	-0.198	1.517
Telcm 87/06*	FF	0.491	0.041	4.125	17.012	-0.004	2.966
	FF	0.018	0.303	1.103	10.312	-0.194	-1.449
Telcm 68/87	FF	0.543	0.279	4.225	17.852	0.139	-0.158
Telcm 68/87*	FF	0.439	0.036	3.597	12.941	0.001	3.043
	FF	0.104	0.243	0.628	4.911	0.139	-3.201

Table 10 Monte Carlo Simulation results for Utilities portfolio for Single factor CAPM and Fama French CAPM before and after October 1987. (*indicates Monte Carlo Simulation results)

Monte Carlo Simulation results Single Factor CAPM before and after October 1987							
Portfolio	Model	Mean		Std. Deviation	Variance	Skewness	Kurtosis
		Statistic	Std. Error	Statistic	Statistic	Statistic	Statistic
Utils 87/06	SF	0.633	0.256	3.884	15.084	-0.331	0.593
Utils 87/06*	SF	0.626	0.013	1.323	1.749	0.045	3.024
	SF	0.007	0.243	2.561	13.335	-0.376	-2.432
Utils 68/87	SF	0.311	0.288	4.365	19.051	0.348	1.147
Utils 68/87*	SF	0.357	0.033	3.251	10.568	0.000	3.023
	SF	-0.046	0.255	1.114	8.483	0.347	-1.876
Monte Carlo Simulation results Fama French CAPM before and after October 1987							
Portfolio	Model	Mean		Std. Deviation	Variance	Skewness	Kurtosis
		Statistic	Std. Error	Statistic	Statistic	Statistic	Statistic
Utils 87/06	FF	0.633	0.256	3.884	15.084	-0.331	0.593
Utils 87/06*	FF	0.617	0.034	3.370	11.356	0.004	2.926
	FF	0.016	0.222	0.514	3.727	-0.335	-2.333
Utils 68/87	FF	0.311	0.288	4.365	19.051	0.348	1.147
Utils 68/87*	FF	0.303	0.042	4.166	17.352	-0.052	2.929
	FF	0.008	0.246	0.199	1.699	0.400	-1.782

Table 11 Monte Carlo Simulation results for Shops portfolio for Single factor CAPM and Fama French CAPM before and after October 1987. (*indicates Monte Carlo Simulation results)

Monte Carlo Simulation results Single Factor CAPM before and after October 1987							
Portfolio	Model	Mean		Std. Deviation	Variance	Skewness	Kurtosis
		Statistic	Std. Error	Statistic	Statistic	Statistic	Statistic
Shops 87/06	SF	0.743	0.312	4.738	22.452	-0.178	0.314
Shops 87/07*	SF	0.736	0.038	3.810	14.518	-0.018	3.026
	SF	0.007	0.274	0.928	7.934	-0.159	-2.712
Shops 68/87	SF	0.491	0.389	5.901	34.823	0.186	1.307
Shops 68/87*	SF	0.434	0.052	5.202	27.056	0.013	3.056
	SF	0.057	0.337	0.700	7.766	0.173	-1.749
Monte Carlo Simulation results Fama French CAPM before and after October 1987							
Portfolio	Model	Mean		Std. Deviation	Variance	Skewness	Kurtosis
		Statistic	Std. Error	Statistic	Statistic	Statistic	Statistic
Shops 87/06	FF	0.743	0.312	4.738	22.452	-0.178	0.314
Shops 87/06*	FF	0.746	0.041	4.114	16.924	0.009	2.918
	FF	-0.003	0.271	0.624	5.528	-0.187	-2.605
Shops 68/87	FF	0.491	0.389	5.901	34.823	0.186	1.307
Shops 68/87*	FF	0.566	0.050	4.991	24.911	0.007	2.996
	FF	-0.075	0.339	0.910	9.912	0.179	-1.688

Table 12 Monte Carlo Simulation results for Health portfolio for Single factor CAPM and Fama French CAPM before and after October 1987. (*indicates Monte Carlo Simulation results)

Monte Carlo Simulation results Single Factor CAPM before and after October 1987							
Portfolio	Model	Mean		Std. Deviation	Variance	Skewness	Kurtosis
		Statistic	Std. Error	Statistic	Statistic	Statistic	Statistic
Hlth 87/06	SF	0.781	0.298	4.514	20.374	-0.024	0.504
Hlth 87/06*	SF	0.764	0.029	2.915	8.495	0.031	2.934
	SF	0.017	0.268	1.599	11.880	-0.055	-2.431
Hlth 68/87	SF	0.477	0.361	5.469	29.909	0.382	3.134
Hlth 68/87*	SF	0.441	0.044	4.379	19.178	-0.028	2.934
	SF	0.036	0.317	1.090	10.731	0.410	0.200
Monte Carlo Simulation results Fama French CAPM before and after October 1987							
Portfolio	Model	Mean		Std. Deviation	Variance	Skewness	Kurtosis
		Statistic	Std. Error	Statistic	Statistic	Statistic	Statistic
Hlth 87/06	FF	0.781	0.298	4.514	20.374	-0.024	0.504
Hlth 87/06*	FF	0.774	0.031	3.106	9.648	0.010	2.933
	FF	0.008	0.267	1.408	10.726	-0.034	-2.429
Hlth 68/87	FF	0.477	0.361	5.469	29.909	0.382	3.134
Hlth 68/87*	FF	0.439	0.044	4.421	19.547	0.015	2.948
	FF	0.038	0.316	1.048	10.362	0.367	0.186

Table 13 Monte Carlo Simulation results for Money portfolio for Single factor CAPM and Fama French CAPM before and after October 1987. (*indicates Monte Carlo Simulation results)

Monte Carlo Simulation results Single Factor CAPM before and after October 1987							
Portfolio	Model	Mean		Std. Deviation	Variance	Skewness	Kurtosis
		Statistic	Std. Error	Statistic	Statistic	Statistic	Statistic
Money 87/06	SF	0.979	0.317	4.807	23.108	-0.418	2.344
Money 87/06*	SF	0.994	0.039	3.874	15.010	-0.015	2.971
	SF	-0.015	0.278	0.933	8.098	-0.403	-0.627
Money 68/87	SF	0.431	0.357	5.421	29.387	0.049	0.456
Money 68/87*	SF	0.455	0.050	5.036	25.361	-0.031	3.023
	SF	-0.023	0.307	0.385	4.025	0.080	-2.567
Monte Carlo Simulation results Fama French CAPM before and after October 1987							
Portfolio	Model	Mean		Std. Deviation	Variance	Skewness	Kurtosis
		Statistic	Std. Error	Statistic	Statistic	Statistic	Statistic
Money 87/06	FF	0.979	0.317	4.807	23.108	-0.418	2.344
Money 87/06*	FF	0.976	0.051	5.140	26.424	0.009	2.958
	FF	0.003	0.266	-0.333	-3.316	-0.427	-0.614
Money 68/87	FF	0.431	0.357	5.421	29.387	0.049	0.456
Money 68/87*	FF	0.397	0.051	5.148	26.500	0.001	3.055
	FF	0.035	0.306	0.273	2.886	0.048	-2.599

Table 14 Monte Carlo Simulation results for Other portfolio for Single factor CAPM and Fama French CAPM before and after October 1987. (*indicates Monte Carlo Simulation results)

Monte Carlo Simulation results Single Factor CAPM before and after October 1987							
Portfolio	Model	Mean		Std. Deviation	Variance	Skewness	Kurtosis
		Statistic	Std. Error	Statistic	Statistic	Statistic	Statistic
Other 87/06	SF	0.444	0.298	4.526	20.482	-0.586	1.527
Other 87/06*	SF	0.475	0.041	4.079	16.639	0.005	2.939
	SF	-0.031	0.258	0.447	3.843	-0.592	-1.412
Other 68/87	SF	0.347	0.394	5.979	35.744	-0.071	0.362
Other 68/87*	SF	0.348	0.057	5.656	31.990	-0.028	2.921
	SF	0.000	0.338	0.323	3.754	-0.043	-2.559
Monte Carlo Simulation results Fama French CAPM before and after October 1987							
Portfolio	Model	Mean		Std. Deviation	Variance	Skewness	Kurtosis
		Statistic	Std. Error	Statistic	Statistic	Statistic	Statistic
Other 87/06	FF	0.444	0.298	4.526	20.482	-0.586	1.527
Other 87/06*	FF	0.422	0.044	4.435	19.673	-0.002	2.997
	FF	0.022	0.254	0.090	0.810	-0.584	-1.470
Other 68/87	FF	0.347	0.394	5.979	35.744	-0.071	0.362
Other 68/87*	FF	0.392	0.054	5.353	28.651	0.002	2.955
	FF	-0.045	0.341	0.626	7.093	-0.073	-2.593

Table 15 Descriptive Statistics 1968 - 1987

Descriptive Statistics 1968 - 1987													
	N	Range	Minimum	Maximum	Sum	Mean		Std. Deviation	Variance	Skewness		Kurtosis	
	Statistic	Statistic	Statistic	Statistic	Statistic	Statistic	Std. Error	Statistic	Statistic	Statistic	Std. Error	Statistic	Std. Error
Mkt-RF	230	29.28	-13.23	16.05	80.35	0.3493	0.3112	4.7195	22.2741	0.0276	0.1605	0.6659	0.3196
SMB	230	20.84	-9.95	10.89	41.25	0.1793	0.1935	2.9343	8.6099	0.2410	0.1605	1.3762	0.3196
HML	230	18.14	-9.77	8.37	120.63	0.5245	0.1833	2.7800	7.7281	-0.1321	0.1605	1.0858	0.3196
NoDur	230	32.96	-14.79	18.17	135.99	0.5913	0.3199	4.8522	23.5443	0.0230	0.1605	1.1657	0.3196
Durbl	230	37.67	-16.69	20.98	97.53	0.4240	0.3659	5.5497	30.7994	0.3334	0.1605	0.9214	0.3196
Manuf	230	31.32	-14.63	16.69	74.48	0.3238	0.3514	5.3293	28.4018	0.1244	0.1605	0.5626	0.3196
Enrgy	230	42.55	-18.97	23.58	125.88	0.5473	0.3946	5.9837	35.8045	0.2085	0.1605	1.1792	0.3196
Chems	230	35.29	-15.58	19.71	98.1	0.4265	0.3299	5.0037	25.0367	0.2991	0.1605	1.2053	0.3196
BusEq	230	35.4	-17.68	17.72	42.08	0.1830	0.3889	5.8978	34.7845	0.2633	0.1605	0.5352	0.3196
Telcm	230	21.39	-9.75	11.64	124.84	0.5428	0.2786	4.2251	17.8518	0.1395	0.1605	-0.1580	0.3196
Utils	230	31.24	-13.03	18.21	71.49	0.3108	0.2878	4.3647	19.0508	0.3475	0.1605	1.1474	0.3196
Shops	230	45.44	-19.3	26.14	112.87	0.4907	0.3891	5.9011	34.8227	0.1859	0.1605	1.3073	0.3196
Hlth	230	45.33	-16.36	28.97	109.72	0.4770	0.3606	5.4689	29.9090	0.3815	0.1605	3.1339	0.3196
Money	230	33.27	-13.22	20.05	99.24	0.4315	0.3574	5.4209	29.3865	0.0491	0.1605	0.4558	0.3196
Other	230	36.74	-18.15	18.59	79.87	0.3473	0.3942	5.9786	35.7437	-0.0714	0.1605	0.3622	0.3196

Table 16 Descriptive Statistics 1987 – 2006

Descriptive Statistics 1987 - 2006													
	N	Range	Minimum	Maximum	Sum	Mean		Std. Deviation	Variance	Skewness		Kurtosis	
	Statistic	Statistic	Statistic	Statistic	Statistic	Statistic	Std. Error	Statistic	Statistic	Statistic	Std. Error	Statistic	Std. Error
Mkt-RF	230	26.5	-16.2	10.3	152.74	0.6641	0.2678	4.0614	16.4946	-0.6578	0.1605	1.0613	0.3196
SMB	230	38.45	-16.58	21.87	35.64	0.1550	0.2331	3.5353	12.4983	0.8326	0.1605	7.9162	0.3196
HML	230	26.37	-12.66	13.71	88.31	0.3840	0.2130	3.2298	10.4315	0.1180	0.1605	3.0202	0.3196
NoDur	230	27.7	-13.46	14.24	174.09	0.7569	0.2582	3.9159	15.3345	-0.1578	0.1605	1.0212	0.3196
Durbl	230	34.86	-19.81	15.05	113.19	0.4921	0.3802	5.7666	33.2538	-0.2860	0.1605	0.7179	0.3196
Manuf	230	28.04	-16.22	11.82	194.18	0.8443	0.3009	4.5636	20.8261	-0.4934	0.1605	0.9873	0.3196
Enrgy	230	30.99	-12.01	18.98	200.42	0.8714	0.3164	4.7981	23.0217	0.3911	0.1605	0.9747	0.3196
Chems	230	25.16	-13.1	12.06	158.21	0.6879	0.2762	4.1881	17.5405	-0.2063	0.1605	0.4625	0.3196
BusEq	230	46.58	-26.37	20.21	171.98	0.7477	0.5021	7.6151	57.9896	-0.2335	0.1605	0.7956	0.3196
Telcm	230	37.29	-16.06	21.23	117.03	0.5088	0.3447	5.2273	27.3243	-0.1979	0.1605	1.5166	0.3196
Utils	230	23.52	-12.45	11.07	145.66	0.6333	0.2561	3.8838	15.0837	-0.3305	0.1605	0.5929	0.3196
Shops	230	27.57	-14.58	12.99	170.83	0.7427	0.3124	4.7383	22.4520	-0.1775	0.1605	0.3138	0.3196
Hlth	230	28.72	-12.66	16.06	179.68	0.7812	0.2976	4.5138	20.3745	-0.0244	0.1605	0.5035	0.3196
Money	230	38.6	-22.41	16.19	225.15	0.9789	0.3170	4.8071	23.1080	-0.4183	0.1605	2.3439	0.3196
Other	230	32.48	-19.57	12.91	102.23	0.4445	0.2984	4.5257	20.4823	-0.5864	0.1605	1.5269	0.3196

Table 17 Single Factor CAPM Regression Results 1968 – 1987

Single Factor CAPM 1968 - 1987										
		Unstandardized Coefficients		t	Sig.	95% Confidence Interval for B		R Square	Adjusted R Square	Std. Error of the Estimate
		B	Std. Error			Lower Bound	Upper Bound			
NoDur	(Constant)	0.264	0.132	1.995	0.047	0.003	0.525	0.8307	0.8300	2.0009
	Mkt-RF	0.937	0.028	33.447	0.000	0.882	0.992			
Durbl	(Constant)	0.074	0.192	0.384	0.701	-0.305	0.452	0.7270	0.7258	2.9062
	Mkt-RF	1.003	0.041	24.639	0.000	0.922	1.083			
Manuf	(Constant)	-0.054	0.100	-0.542	0.588	-0.252	0.143	0.9193	0.9189	1.5175
	Mkt-RF	1.083	0.021	50.954	0.000	1.041	1.125			
Enrgy	(Constant)	0.224	0.271	0.827	0.409	-0.310	0.758	0.5326	0.5306	4.0996
	Mkt-RF	0.925	0.057	16.120	0.000	0.812	1.038			
Chems	(Constant)	0.084	0.127	0.665	0.507	-0.165	0.334	0.8540	0.8533	1.9162
	Mkt-RF	0.980	0.027	36.516	0.000	0.927	1.033			
BusEq	(Constant)	-0.201	0.186	-1.078	0.282	-0.568	0.166	0.7729	0.7719	2.8170
	Mkt-RF	1.099	0.039	27.853	0.000	1.021	1.176			
Telcm	(Constant)	0.324	0.200	1.619	0.107	-0.070	0.718	0.4890	0.4868	3.0269
	Mkt-RF	0.626	0.042	14.772	0.000	0.543	0.710			
Utils	(Constant)	0.070	0.193	0.364	0.716	-0.310	0.451	0.5547	0.5527	2.9191
	Mkt-RF	0.689	0.041	16.852	0.000	0.608	0.769			
Shops	(Constant)	0.103	0.180	0.572	0.568	-0.252	0.458	0.7875	0.7865	2.7265
	Mkt-RF	1.110	0.038	29.064	0.000	1.034	1.185			
Hlth	(Constant)	0.152	0.216	0.704	0.482	-0.274	0.579	0.6433	0.6417	3.2736
	Mkt-RF	0.929	0.046	20.277	0.000	0.839	1.020			
Money	(Constant)	0.062	0.141	0.443	0.658	-0.215	0.340	0.8459	0.8453	2.1324
	Mkt-RF	1.056	0.030	35.384	0.000	0.998	1.115			
Other	(Constant)	-0.073	0.125	-0.583	0.560	-0.319	0.173	0.9008	0.9004	1.8868
	Mkt-RF	1.202	0.026	45.511	0.000	1.150	1.254			

Table 18 Single Factor CAPM Regression Results 1987 – 2006

Single Factor CAPM 1987 - 2006										
		Coefficients		t	Sig.	95% Confidence		R Square	Adjusted R Square	Std. Error of the Estimate
		B	Std. Error			Lower Bound	Upper Bound			
NoDur	(Constant)	0.334	0.197	1.696	0.091	-0.054	0.721	0.4370	0.4345	2.9448
	Mkt-RF	0.637	0.048	13.302	0.000	0.543	0.732			
Durbl	(Constant)	-0.180	0.271	-0.667	0.505	-0.714	0.353	0.5089	0.5067	4.0502
	Mkt-RF	1.013	0.066	15.370	0.000	0.883	1.143			
Manuf	(Constant)	0.186	0.144	1.292	0.198	-0.098	0.469	0.7783	0.7774	2.1532
	Mkt-RF	0.991	0.035	28.295	0.000	0.922	1.060			
Engry	(Constant)	0.487	0.280	1.739	0.083	-0.065	1.039	0.2397	0.2363	4.1930
	Mkt-RF	0.578	0.068	8.478	0.000	0.444	0.713			
Chems	(Constant)	0.209	0.201	1.044	0.298	-0.186	0.605	0.4881	0.4859	3.0030
	Mkt-RF	0.720	0.049	14.745	0.000	0.624	0.817			
BusEq	(Constant)	-0.314	0.267	-1.177	0.241	-0.839	0.212	0.7267	0.7255	3.9900
	Mkt-RF	1.598	0.065	24.620	0.000	1.470	1.726			
Telcm	(Constant)	-0.147	0.225	-0.654	0.514	-0.589	0.296	0.5885	0.5867	3.3606
	Mkt-RF	0.987	0.055	18.057	0.000	0.880	1.095			
Utils	(Constant)	0.417	0.245	1.706	0.089	-0.065	0.899	0.1158	0.1119	3.6601
	Mkt-RF	0.325	0.060	5.463	0.000	0.208	0.443			
Shops	(Constant)	0.118	0.188	0.629	0.530	-0.252	0.488	0.6502	0.6486	2.8087
	Mkt-RF	0.941	0.046	20.585	0.000	0.851	1.031			
Hlth	(Constant)	0.305	0.231	1.320	0.188	-0.150	0.760	0.4167	0.4141	3.4549
	Mkt-RF	0.717	0.056	12.762	0.000	0.607	0.828			
Money	(Constant)	0.352	0.194	1.813	0.071	-0.031	0.734	0.6367	0.6351	2.9039
	Mkt-RF	0.944	0.047	19.988	0.000	0.851	1.038			
Other	(Constant)	-0.225	0.130	-1.734	0.084	-0.480	0.031	0.8173	0.8165	1.9386
	Mkt-RF	1.007	0.032	31.939	0.000	0.945	1.070			

Table 19 Fama French CAPM Regression Results 1968 – 1987

Fama French CAPM 1968 - 1987										
		Coefficients		t	Sig.	95% Confidence		R Square	Adjusted R Square	Std. Error of the Estimate
		B	Std. Error			Lower Bound	Upper Bound			
NoDur	(Constant)	0.213	0.134	1.588	0.114	-0.051	0.476	0.8372	0.8351	1.9706
	Mkt-RF	0.922	0.032	28.853	0.000	0.859	0.985			
	SMB	0.128	0.048	2.670	0.008	0.033	0.222			
	HML	0.064	0.051	1.270	0.205	-0.035	0.164			
Durbl	(Constant)	-0.080	0.193	-0.417	0.677	-0.460	0.299	0.7423	0.7389	2.8357
	Mkt-RF	1.032	0.046	22.442	0.000	0.941	1.123			
	SMB	0.104	0.069	1.508	0.133	-0.032	0.239			
	HML	0.239	0.073	3.277	0.001	0.095	0.383			
Manuf	(Constant)	-0.067	0.102	-0.655	0.513	-0.268	0.134	0.9215	0.9205	1.5027
	Mkt-RF	1.063	0.024	43.605	0.000	1.015	1.111			
	SMB	0.093	0.036	2.542	0.012	0.021	0.165			
	HML	0.005	0.039	0.139	0.889	-0.071	0.081			
Enrgy	(Constant)	0.209	0.266	0.785	0.433	-0.316	0.734	0.5759	0.5703	3.9224
	Mkt-RF	1.051	0.064	16.528	0.000	0.926	1.177			
	SMB	-0.451	0.095	-4.741	0.000	-0.639	-0.264			
	HML	0.099	0.101	0.980	0.328	-0.100	0.297			
Chems	(Constant)	0.099	0.128	0.770	0.442	-0.154	0.351	0.8597	0.8579	1.8865
	Mkt-RF	1.012	0.031	33.067	0.000	0.951	1.072			
	SMB	-0.139	0.046	-3.038	0.003	-0.229	-0.049			
	HML	-0.001	0.048	-0.021	0.983	-0.097	0.095			
BusEq	(Constant)	0.009	0.179	0.050	0.960	-0.343	0.361	0.8038	0.8012	2.6299
	Mkt-RF	0.979	0.043	22.955	0.000	0.895	1.063			
	SMB	0.158	0.064	2.468	0.014	0.032	0.283			
	HML	-0.374	0.068	-5.537	0.000	-0.507	-0.241			
Telcm	(Constant)	0.142	0.197	0.723	0.471	-0.245	0.529	0.5370	0.5308	2.8941
	Mkt-RF	0.735	0.047	15.653	0.000	0.642	0.827			
	SMB	-0.154	0.070	-2.199	0.029	-0.293	-0.016			
	HML	0.328	0.074	4.404	0.000	0.181	0.474			
Utils	(Constant)	-0.194	0.179	-1.085	0.279	-0.547	0.159	0.6404	0.6357	2.6345
	Mkt-RF	0.834	0.043	19.524	0.000	0.750	0.918			
	SMB	-0.178	0.064	-2.785	0.006	-0.304	-0.052			
	HML	0.468	0.068	6.912	0.000	0.335	0.602			
Shops	(Constant)	0.073	0.177	0.413	0.680	-0.276	0.423	0.8068	0.8043	2.6107
	Mkt-RF	1.040	0.042	24.571	0.000	0.957	1.124			
	SMB	0.301	0.063	4.757	0.000	0.177	0.426			
	HML	0.000	0.067	0.004	0.997	-0.132	0.132			
Hlth	(Constant)	0.535	0.191	2.806	0.005	0.159	0.911	0.7398	0.7363	2.8082
	Mkt-RF	0.853	0.046	18.738	0.000	0.764	0.943			
	SMB	-0.245	0.068	-3.601	0.000	-0.380	-0.111			
	HML	-0.595	0.072	-8.248	0.000	-0.738	-0.453			
Money	(Constant)	-0.064	0.140	-0.459	0.647	-0.341	0.212	0.8568	0.8549	2.0651
	Mkt-RF	1.081	0.033	32.283	0.000	1.015	1.147			
	SMB	0.083	0.050	1.664	0.097	-0.015	0.182			
	HML	0.197	0.053	3.706	0.000	0.092	0.301			
Other	(Constant)	-0.093	0.093	-0.999	0.319	-0.276	0.090	0.9482	0.9475	1.3701
	Mkt-RF	1.082	0.022	48.720	0.000	1.039	1.126			
	SMB	0.477	0.033	14.352	0.000	0.412	0.543			
	HML	-0.045	0.035	-1.271	0.205	-0.114	0.025			

Table 20 Fama French CAPM Regression Results 1987 – 2006

Fama French CAPM 1987 - 2006										
		Coefficients		t	Sig.	95% Confidence		R Square	Adjusted R Square	Std. Error of the Estimate
		B	Std. Error			Lower Bound	Upper Bound			
NoDur	(Constant)	0.164	0.187	0.879	0.380	-0.204	0.532	0.5275	0.5212	2.7096
	Mkt-RF	0.777	0.051	15.250	0.000	0.676	0.877			
	SMB	-0.185	0.056	-3.296	0.001	-0.295	-0.074			
	HML	0.275	0.070	3.945	0.000	0.137	0.412			
Durbl	(Constant)	-0.717	0.239	-2.997	0.003	-1.188	-0.246	0.6429	0.6382	3.4688
	Mkt-RF	1.307	0.065	20.039	0.000	1.178	1.435			
	SMB	0.184	0.072	2.560	0.011	0.042	0.325			
	HML	0.815	0.089	9.142	0.000	0.639	0.990			
Manuf	(Constant)	-0.023	0.138	-0.164	0.870	-0.295	0.249	0.8100	0.8075	2.0023
	Mkt-RF	1.102	0.038	29.278	0.000	1.028	1.176			
	SMB	0.091	0.041	2.184	0.030	0.009	0.172			
	HML	0.315	0.051	6.134	0.000	0.214	0.417			
Enrgy	(Constant)	0.117	0.268	0.436	0.663	-0.411	0.645	0.3526	0.3440	3.8862
	Mkt-RF	0.807	0.073	11.050	0.000	0.663	0.951			
	SMB	-0.008	0.080	-0.100	0.920	-0.167	0.150			
	HML	0.572	0.100	5.734	0.000	0.376	0.769			
Chems	(Constant)	0.008	0.191	0.044	0.965	-0.368	0.384	0.5695	0.5638	2.7660
	Mkt-RF	0.872	0.052	16.774	0.000	0.770	0.975			
	SMB	-0.147	0.057	-2.576	0.011	-0.260	-0.035			
	HML	0.321	0.071	4.516	0.000	0.181	0.461			
BusEq	(Constant)	0.188	0.215	0.871	0.384	-0.237	0.612	0.8340	0.8318	3.1229
	Mkt-RF	1.258	0.059	21.423	0.000	1.142	1.373			
	SMB	0.172	0.065	2.668	0.008	0.045	0.300			
	HML	-0.786	0.080	-9.798	0.000	-0.944	-0.628			
Telcm	(Constant)	-0.080	0.225	-0.354	0.724	-0.523	0.364	0.6157	0.6106	3.2620
	Mkt-RF	0.997	0.061	16.266	0.000	0.877	1.118			
	SMB	-0.267	0.067	-3.956	0.000	-0.400	-0.134			
	HML	-0.085	0.084	-1.010	0.314	-0.250	0.080			
Utils	(Constant)	-0.026	0.209	-0.124	0.902	-0.437	0.385	0.4010	0.3930	3.0258
	Mkt-RF	0.615	0.057	10.805	0.000	0.502	0.727			
	SMB	-0.090	0.063	-1.444	0.150	-0.214	0.033			
	HML	0.690	0.078	8.880	0.000	0.537	0.843			
Shops	(Constant)	0.007	0.192	0.037	0.970	-0.371	0.385	0.6600	0.6555	2.7812
	Mkt-RF	1.008	0.052	19.275	0.000	0.905	1.111			
	SMB	0.006	0.058	0.096	0.923	-0.108	0.119			
	HML	0.171	0.071	2.389	0.018	0.030	0.311			
Hlth	(Constant)	0.481	0.226	2.132	0.034	0.036	0.926	0.4806	0.4738	3.2745
	Mkt-RF	0.676	0.062	10.990	0.000	0.555	0.798			
	SMB	-0.351	0.068	-5.182	0.000	-0.485	-0.218			
	HML	-0.248	0.084	-2.943	0.004	-0.413	-0.082			
Money	(Constant)	0.003	0.153	0.021	0.983	-0.299	0.306	0.7885	0.7857	2.2253
	Mkt-RF	1.193	0.042	28.511	0.000	1.110	1.275			
	SMB	-0.179	0.046	-3.885	0.000	-0.270	-0.088			
	HML	0.551	0.057	9.633	0.000	0.438	0.663			
Other	(Constant)	-0.366	0.128	-2.858	0.005	-0.619	-0.114	0.8336	0.8314	1.8582
	Mkt-RF	1.070	0.035	30.638	0.000	1.001	1.139			
	SMB	0.126	0.038	3.281	0.001	0.050	0.202			
	HML	0.210	0.048	4.393	0.000	0.116	0.304			

Table 21 Industry definitions using four-digit SIC Codes¹

INDUSTRY DEFINITIONS	
NoDur Consumer NonDurables -- Food, Tobacco, Textiles, Apparel, Leather, Toys	0100-0999 2000-2399 2700-2749 2770-2799 3100-3199 3940-3989
2 Durbl Consumer Durables -- Cars, TV's, Furniture, Household Appliances	2500-2519 2590-2599 3630-3659 3710-3711 3714-3714 3716-3716 3750-3751 3792-3792 3900-3939 3990-3999
3 Manuf Manufacturing -- Machinery, Trucks, Planes, Off Furn, Paper, Com Printing	2520-2589 2600-2699 2750-2769 3000-3099 3200-3569 3580-3629 3700-3709 3712-3713 3715-3715 3717-3749 3752-3791 3793-3799 3830-3839 3860-3899
4 Enrgy Oil, Gas, and Coal Extraction and Products	1200-1399 2900-2999
5 Chems Chemicals and Allied Products	2800-2829 2840-2899
6 BusEq Business Equipment -- Computers, Software, and Electronic Equipment	3570-3579 3660-3692 3694-3699 3810-3829 7370-7379
7 Telcm Telephone and Television Transmission	4800-4899
8 Utils Utilities	4900-4949
9 Shops Wholesale, Retail, and Some Services (Laundries, Repair Shops)	5000-5999 7200-7299 7600-7699
10 Hlth Healthcare, Medical Equipment, and Drugs	2830-2839 3693-3693 3840-3859 8000-8099
11 Money Finance	6000-6999
12 Other Other -- Mines, Constr, BldMt, Trans, Hotels, Bus Serv, Entertainment	

¹ http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/Data_Library/changes_ind.html