MODELLING AND SIMULATION OF PAPER STRUCTURE DEVELOPMENT

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ABSTRACT

A numerical tool has been developed for particle-level simulations of fibre suspension flows, particularly forming of the fibre network structure of paper sheets in the paper machine. The model considers inert fibres of various equilibrium shapes, and finite stiffness, interacting with each other through normal, frictional, and lubrication forces, and with the surrounding fluid medium through hydrodynamic forces. Fibre-fluid interactions in the non-creeping flow regime are taken into account, and the two-way coupling between the solids and the fluid phases is included by enforcing momentum conservation between phases. The incompressible three-dimensional Navier–Stokes equations are employed to model the motion of the fluid medium.

The validity of the model has been tested by comparing simulation results with experimental data from the literature. It was demonstrated that the model predicts well the motion of isolated fibres in shear flow over a wide range of fibre flexibilities. It was also shown that the model predicts details of the orientation distribution of multiple, straight, rigid fibres in a sheared suspension. Furthermore, model predictions of the shear viscosity and first normal stress difference were in fair agreement with experimental data found in the literature. Since the model is based solely on first principles physics, quantitative predictions could be made without any parameter fitting.

Based on these validations, a series of simulations have been performed to investigate the basic mechanisms responsible for the development of the stress tensor components for monodispersed, non-Brownian fibres suspended in a Newtonian fluid in shear flow. The effects of fibre aspect ratio, concentration, and inter-particle friction, as well as the tendency of fibre agglomeration, were examined in the non-concentrated regimes. For the case of well dispersed suspensions, semi-empirical relationships were found between the aforementioned fibre suspension properties, and the steady state apparent shear viscosity, and the first/second normal stress differences.

Finally, simulations have been conducted for the development of paper structures in the forming section of the paper machine. The conditions used for the simulations were retrieved from pilot-scale forming trial data in the literature, and from real pulp fibre analyses. Dewatering was simulated by moving two forming fabrics toward each other through a fibre suspension. Effects of the jet-to-wire speed difference on the fibre orientation anisotropy, the mass density distribution, and three-dimensionality of the fibre network, were investigated. Simulation results showed
that the model captures well the essential features of the forming effects on these paper structure parameters, and also posed new questions on the conventional wisdom of the forming mechanics.

Keywords: forming, fibre, paper, fibre suspension, paper structure, simulation, rheology
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LIST OF PAPERS

This thesis is mainly based on the following four papers, herein referred to by their Roman numerals:

Paper I  
*Simulation of the motion of flexible fibres in viscous fluid flow*,  
Stefan B. Lindström and Tetsu Uesaka,  

Paper II  
*Simulation of semidilute suspensions of non-Brownian fibres in shear flow*,  
Stefan B. Lindström and Tetsu Uesaka,  

Paper III  
*Particle-level simulation of forming of the fibre network in papermaking*,  
Stefan B. Lindström and Tetsu Uesaka,  

Paper IV  
*A numerical investigation of the rheology of sheared fibre suspensions*,  
Stefan B. Lindström and Tetsu Uesaka,  
Submitted to Physics of Fluids, 2008.
1. INTRODUCTION

One of the most important end-usages of paper is as a substrate for printing. High quality prints are obtained by controlling the interactions between printing plate, ink, and substrate in the printing press. It is also these interactions that create numerous print quality problems associated with non-uniform structures of paper. For example print-through and show-through are related to a certain type of formation (mass density distribution) and two-sidedness in the porous structure. Mottles in print density and print gloss are often attributed to the variations of surface pore structures and surface fibre orientation distribution. Deletion in electrophotography and missing dots in gravure printing are related to the contact topology of the paper surface. Curl and cockle are other manifestations of the variations of density, composition, and fibre orientation both in the in-plane and out-of-plane directions. It is an outstanding question in the industry how these microstructures of paper are created and influenced by the papermaking unit processes.

The most common approach to try to answer this question may be to conduct trials under controlled process conditions using real or pilot-scale papermaking equipment, and to measure effects on the paper structure with the help of various structure characterisation techniques. If successfully performed, this approach can provide phenomenological relationships between the process variables and the paper structures, and, in fact, a number of important insights have been accumulated so far. However, it is also recognised that, because of the complex, multivariate nature of the papermaking process, the trial results are sometimes inconsistent, not necessarily repeated even under seemingly same process conditions. In addition, most of the experimental techniques currently available do not have enough resolution to detect the length-scales in question (i.e. microstructures in the order of halftone dot size and fibre floc size) under dynamic process conditions. Therefore, the questions regarding the mechanisms behind the development of the paper structures still remain as outstanding.

One way to recreate the microstructures of paper, and to gain understanding of the mechanisms behind their formation, is to systematically model each unit process of the paper machine at a particle level. As a first step, this thesis is concerned with modelling the forming section, which is the first unit process in the paper machine. In order to simulate the forming of the fibre network, a particle-level fibre suspension model, whose validity range includes the flow conditions found in the forming section of modern paper machines, is needed. The intended application imposes extensive and diverse demands on this fibre suspension model. Flexible fibres interacting with each other and the fluid medium must be considered, for finite Reynolds number flows. Consequently, a large part of this thesis is dedicated to the development and validation of a new fibre suspension model.

1.1. Paper and papermaking

In this section, an overview of microscale structures of papers and the paper production process is given, in order to put present work into context, and to elucidate its motivation.
Figure 1. An SEM photograph of the cross section of a common 80 g/m² office paper. Fibres appear as grey, while the denser filler particles are black. Note the disordered character of the microstructure. The width of the image is 380 µm. (Courtesy of A.-L. Lindström)

1.1.1. Paper structure

The most prominent structural component of papers is the fibres. The main body of the paper is a network of fibres mostly aligned in the plane of the paper; fibre lengths are normally greater than the paper thickness. The network structure has a statistical aspect; the structure appears to be random. In reality, however, its structural features are determined in the production process, which makes paper structures deviate from random networks in many fundamental ways.

The fibres of the paper typically have a very wide range of morphological properties. The largest are intact plant cells, their lengths being several millimetres. The smallest are fibre fragments, so called fines. The interested reader is referred to the book *Paper Physics* [1] for an overview on the topic of paper structures.

To alter the optical and mechanical properties of papers, it is common practise to add fillers to the sheet structure. The fillers, e.g., CaCO₃, are fine grained, non-organic powders with high opacity and whiteness. The microvariations in the distribution of fillers near the surface of uncoated papers greatly affect their performance when printed. An SEM image of the cross section of a copy paper is presented in Fig. 1. Fibres appear as grey areas, while the denser filler particles are black.

In addition to fibres, fines, and fillers, a coating layer can be applied to the surface of the paper to decrease surface roughness, increase whiteness, and to improve printability. The bulk of the coating stays on the surface, but a fraction enters the bulk of the paper.

1.1.2. Papermaking

Due to the requirement of production efficiency, paper machines are large pieces of machinery operating at high speeds. Modern machines output a 10 m wide paper sheet at a rate of about 2,000 m/min—a velocity comparable to that of a speeding car (120 km/h).

The raw materials used for papermaking are suspended in water at the inlet of the paper machine. The suspension, known as the stock, contains fibres, fines, and fillers. Small quantities of chemical additives are also introduced. Some of these, the retention aids, are designed to cause particle agglomeration. Thus, the stock con-
tains fibres, fines, fillers, and agglomerates of these components. The paper machine processes the stock in a serial combination of unit processes. Each unit process is described below:

**Forming** The forming section is the first unit process of papermaking. Here, the stock is distributed evenly in a wide thin jet. The jet is then drained from most of its water, producing a wet paper web. A more detailed description of the forming section can be found in Sec. 1.1.3.

**Wet pressing** The wet paper web is pressed in one or multiple nips while resting on a permeable fabric, and additional water is drained.

**Drying** The paper web is transported on steam-heated rolls. The heat is used to evaporate all but a small percentage of water from the sheet.

**Calendering** The paper passes one or several nips that smooth out the surface and makes the paper more compact.

**Coating** Some paper grades are coated. That is, a thin layer of coating suspension is applied to the surface. Coated papers are further dried and calendered a second time.

Once the forming section has created a wet paper web, fibres move little relative to each other, due to geometrical constraints. Consequently, many features of the final geometry are determined in the forming section, and subsequent unit processes modify the geometry by draining the remaining fluid, and consolidating the network structure. This illustrates the importance of accurately predicting the effects of forming, in order to predict the paper structures.

### 1.1.3. Forming section

The forming section is designated for continuously converting the stock at the inlet of the paper machine into a wet paper web. The forming section includes two components: the headbox and the former. The headbox converts the inlet flow into a wide thin jet of fibre suspension. The former drains most of the fluid in the jet through one or two permeable forming fabrics, thus producing a wet paper web. Extensive overviews of paper sheet forming have previously been written by Parker [2], Norman [3, 4], and Attwood [5].

The stock is fed into the headbox, where it is diluted to a solids concentration of 1–10 g/l. Inside the headbox, the suspension passes through a manifold of tubes, designed to introduce turbulence into the suspension for the purpose of mixing. The suspension continues through a contraction, often divided into several narrow channels by thin flexible separation vanes [4]. More or less coherent flow structures are created at the tips of the vanes (see Fig. 2). At the end of the contraction, the headbox nozzle ejects a plane jet whose thickness is typically of centimetre length scale. Considering the high machine speeds and the unstable nature of high Reynolds number fluid flows, the headbox design is critical to paper quality; instabilities in the flow directly affect the in-plane basis weight distribution and the network structures of the paper, such as fibre orientation.
Figure 2. A headbox nozzle equipped with three flexible vanes. The vanes stabilise the flow. At the same time, coherent flow structures are created at their tips, affecting the orientation distribution of fibres in the paper (see Ref. [6]).

Figure 3. Principles of the roll–blade gap former. The headbox ① ejects a plane jet of fibre suspension ② onto the wires. The top wire ③ runs under tension, and thus provides drainage pressure. The bottom wire ④ rests on a permeable roll ⑤. After gentle drainage on the roll, a significant amount of fluid still remains between the wires. A set of blades ⑥ induces pressure pulses in the suspension that intermittently push most of the remaining fluid through the wires. Note that the illustration is out-of-scale.

The jet travels through air until it impinges on the forming fabrics of the former. There are several types of formers, but in this work the roll–blade gap former is especially considered, even though the theories presented herein are applicable to other types of forming equipment as well. A schematic illustration of the roll–blade gap former is provided in Fig. 3. The jet impinges on two forming fabrics, also known as wires. The bottom wire is supported by a permeable roll, and the top wire runs under tension to provide drainage pressure. After the roll, a set of blades induces pressure pulses that intermittently push most of the remaining fluid through the wires. These blades are placed in different configurations on both sides of the wires.

During forming fibres tend to align preferentially in the machine direction [7, 8]. This is partly due to the interactions of the fibres with the elongational flow in the contraction of the headbox [8, 9], and partly to the interactions of the suspended fibres with the wires and the shear flow between the forming fabrics [10, 11]. Depending on the end-use of a paper, different degrees of fibre alignment is desired.

Another property of the fibre suspension that is important during forming is the intensity of flocculation. Fibres tend to agglomerate and form clusters called
flocs [12]. Fibre flocs cause basis weight variations in the final paper, which are detrimental to many paper properties. Both fibre alignment and the formation of flocs are features of the general behaviour of flowing fibre suspensions. Thus, to understand, or to model, the mechanisms behind the development of the microstructures in paper, it is essential to understand the dynamics of fibre suspensions.

1.2. Fibre suspensions

In this section, the field of fibre suspensions flows, and numerical simulation of the same are reviewed.

1.2.1. Fibre suspension flow

The problem of predicting particle motions in a flow velocity gradient was originally investigated theoretically by Einstein [13], who calculated the effect of the presence of spherical beads on the apparent viscosity of a suspension. Jeffery [14] used a similar theoretical approach to derive the motion of isolated ellipsoids in viscous shear flow. His theory predicts that massless ellipsoids show repeatable orbits around the vorticity axis of the flow, and he also gave a description of these orbits in analytical form. The theory has been experimentally confirmed for prolate ellipsoids [15, 16]. It was later discovered by Bretherton [17] that “the orientation of the axis of almost any body of revolution is a periodic function of time in any unidirectional flow”. That is, all bodies of revolution display an orbital behaviour in a linear flow gradient. Traditionally, ideal fibres have been represented by circular cylinders. It has been shown theoretically, by Cox [18, 19], and experimentally, by several researchers [16, 20, 21, 22], that circular cylinders perform Jeffery orbits. The motion of rigid, isolated fibres in a Newtonian fluid is thus relatively well known.

When multiple fibres are dispersed in a viscous medium, their motion, subject to some flow, becomes much more difficult to predict due to the hydrodynamic and mechanical contact interactions between fibres, which disturb the Jeffery orbits [21]. The physics of fibre suspensions depends heavily on the nature and magnitude of the fibre–fibre interactions. Therefore, fibre suspensions can be divided into regimes, in which similar physical behaviour can be observed: the dilute regime ($nL^3 \ll 1$); the semidilute regime ($nL^3 > 1$ and $nL^2D \ll 1$, see Ref. [23]); and the concentrated regime ($nL^2D > 1$). Here, $n$ is the number of fibres per unit volume, $L$ is the fibre length, and $D$ is the fibre diameter. In the dilute regime, most interactions are long range hydrodynamic, which are quite weak. The fibre motion in dilute suspensions is well approximated by Jeffery’s theory for isolated fibres, whereas the theory is less accurate for the semidilute and concentrated regimes.

For a given orientation distribution of freely moving, straight, slender fibres suspended in a Newtonian medium, Batchelor [24] derived the contribution of the fibres to the stress tensor of the suspension, using the slender body theory of Burgers [25] with corrections for the finite aspect ratio. The components of the stress tensor, in turn, render rheological properties of the suspension, such as the viscosity, and the first and second normal stress differences. Shaqfeh and Fredrickson [26] contributed to Batchelor’s theory by including effects of hydrodynamic fibre–fibre interactions in the semidilute regime. Stover et al. [27] and Petrich et al. [28] measured the fibre
orientation distribution in sheared, semidilute suspensions of straight fibres. They observed a reasonable, but not good, agreement between the measured viscosity and the viscosity calculated from the measured orientation distribution using Batchelor’s theory.

Several authors have modelled the effect of the fibre–fibre interactions on the orientation distribution using the rotary diffusivity model derived by Burgers [25]. Leal and Hinch [29] investigated the effects of weak rotary diffusion caused by Brownian motion on the orientation distribution of sheared suspensions. They also compared their theoretical predictions with the orientation distribution of dilute fibre suspensions, as measured by Anczurowski and Mason [30]. They concluded that the experiments did not support their theory, most likely because effects of hydrodynamic interactions dominated in the experimental investigation. The rotary diffusivity model has been used by Shaqfeh and Koch [31] to predict the orientation distribution of axisymmetric particles flowing through a fixed bed of cylinders or spheres—results that were confirmed experimentally by Frattini et al. [32]. Rahnama et al. [33] introduced an anisotropic rotary diffusivity, with orientation dependence of the diffusivities, to model dilute and semidilute fibre suspensions in simple shear flow. The theoretical orientation distribution agreed with the experimentally obtained distributions of Stover et al. [27], but only after parameter fitting in each instance of the experiments. The validity of the rotary diffusivity model is, however, restricted to concentration regimes where effects of surface contact interactions are negligible.

In summary, only the properties of dilute and semidilute suspensions of straight, rigid, slender, massless fibres in a Newtonian fluid might be said to be adequately understood. It has been difficult to theoretically obtain quantitative predictions of the orientation distribution or the rheological properties of semidilute and concentrated suspensions, for fibres of a finite aspect ratio or a finite stiffness.

1.2.2. Simulation of fibre suspension flow

Due to the inherent complexity of fibre suspension flows, particle-level simulations may be a useful complement to theoretical and experimental investigations. Because of the exceedingly high computational demands of Direct Numerical Simulations (DNS) of fibre suspensions, many researchers have proposed fibre suspension models, in which the fibres are represented by multi-rigid-body systems of hydrodynamically simple particles, for instance, interconnected spheres or ellipsoids. Such approaches also make possible the modelling of fibres with various equilibrium shapes and a finite stiffness.

Fan et al. [34] used Stokesian Dynamics [35] to model the motion of rigid fibres in a sheared suspension. The fibres were modelled as cylinders, interacting with each other through lubrication forces. Long range hydrodynamic interactions were accounted for using Stokesian Dynamics with a slender body approximation. Mechanical contact interactions were neglected. Fan et al. reasonably predicted the viscosity in the dilute and the semidilute regime using the theoretical framework of Batchelor [24]. The predicted fibre orientation distribution was, however, not in good agreement with the experimental results of Stover et al. [27]. Sundararajakumar and Koch [36] performed the same type of numerical investigation taking only mechanical contact interactions into account. Their simulations predicted the development of a nonzero first normal stress difference for increasing concentrations.
Their model also predicted great variations of the fibre orientation distribution with concentration. This is not consistent with the later experimental observations of Petrich et al. [28], which showed that concentration only have a small effect on the fibre orientation distribution.

Yamamoto and Matsuoka [37] modelled flexible fibres as chains of interconnected spherical beads, interacting with a prescribed fluid flow through viscous drag forces, and with each other through short range hydrodynamic forces, and distance-dependent normal forces to prevent surface overlaps. A similar model was employed by Joung et al. [38, 39], who included a restoring moment acting to straighten the fibres as they bend due to the interactions with the flow. Joung et al. also used Stokesian Dynamics [35] to include long range interactions between fibres. However, since the force acting on each spherical bead must be calculated as the sum of the interactions with all other beads of the suspension, this approach is still quite numerically demanding for a large number of fibres.

Schmid et al. [40] used a chain of interconnected rods instead of spherical beads. This way, the number of solid bodies of each fibre was reduced, thus improving the numerical efficiency. In their model, massless fibres interact with the fluid through viscous drag forces, and with each other through normal and frictional forces; hydrodynamic fibre–fibre interactions and self-interactions were neglected. Schmid et al. used their model to study the flocculation of fibres in sheared suspensions. The validity of the model is restricted to creeping flow conditions. With this model, it is difficult to simulate the motion of stiff fibres or fibres of many fibre segments, because in those cases, a very short time step must be chosen to ensure numerical stability. Even so, it is noteworthy that Switzer et al. [42] successfully used the model to simulate the self-filtration of fibres into a wet web.

Using the Immersed Boundary Method [43], Stockie and Green [44] modelled the motion of isolated, massless, flexible fibres in shear flow, in two dimensions. The fibres were modelled as chains of line segments, and thus similar to the fibre model of Schmid et al. [40]. However, Stockie and Green took into account the two-way coupling between the fibre and the fluid phase, and calculated the flow field including the disturbance caused by the fibre. Tornberg and Shelley [45] used the slender body theory of Keller and Rubinow [46] to model the motion of multiple, slender, flexible filaments interacting through long range hydrodynamic forces in a three-dimensional Stokes flow, thus including the effects of the fibres on the fluid, but ignoring short range interactions and effects of the finite fibre aspect ratio. Qi [47] simulated the motion of isolated, flexible fibres moving in a three-dimensional flow field, using the lattice Boltzman method for calculating the flow, and a chain of rods for modelling the fibre. Qi also took particle inertia and the two-way interactions of the fibre and the fluid into account. Only isolated fibres of aspect ratio less than 9 was simulated due to the computational cost involved.

Long range and short range hydrodynamic interactions between fibres, as well as mechanical contact interactions, may affect the fibre suspension flow and the spatial distribution and orientation distribution of the fibres. In all previous attempts to simulate the motion of multiple suspended fibres, at least one of these types of interactions has been neglected. No simulation results have been presented showing that the fibre orientation distribution can be predicted for the semidilute or the

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1 A similar model was proposed by Ning and Melrose [41].
concentrated regimes. The orientation distribution is poorly predicted when mechanical contact interactions are neglected [34], and equally poorly predicted when only mechanical contact interactions are taken into account (compare the simulations of Sundararajakumar and Koch [36] with experimental results [27, 28, 30]). In the present situation, it is difficult to neglect any type of hydrodynamic or mechanical contact interactions. At the same time, although good accuracy for isolated fibre motion prediction was achieved by Qi [47] with his DNS approach, that method severely restricts the range of fibre aspect ratios and the number of fibres that can be modelled simultaneously, due to the computational cost.

Many important contributions have been made by the aforementioned researchers to the area of fibre suspension flow simulations; the accuracy and computational efficiency of simulations have improved continuously during the past years. However, the performance of the models is still insufficient for predicting the fibre configuration development of multiple interacting fibres in semidilute and the concentrated flowing suspensions. Consequently, there is still much to be done before simulations can significantly contribute to the understanding of the physics of fibre suspension flows. Moreover, to simulate industrial applications such as papermaking, the validity range of the models must be extended, as will be discussed in the problem analysis section below.

1.3. Problem analysis

The fundamental problem in simulating forming of the paper sheet is modelling the fibre suspension. Before a suitable fibre suspension model can be formulated, we must carefully consider the physics of flowing wood fibre suspensions in papermaking, so that it is possible to identify all potentially relevant phenomena, which must be taken into account.

The suspension structure has some inherent microscopic length scales, namely the fibre length \( L \), the fibre diameter \( D \), and the inter-fibre spacing \((nL)^{-1/2}\). A necessary condition for describing the physics of the system with averaged field equations is that the smallest characteristic length scale \( h \) of macroscopic motion is much larger than the characteristic length scales of the microscopic phenomena:

\[
L \ll h, \quad (1.1a)
\]
\[
D \ll h, \quad (1.1b)
\]
\[
(nL)^{-1/2} \ll h. \quad (1.1c)
\]

In paper machine forming, the fibre length and jet thickness are of the same order, rendering at least relation (1.1a) false. Consequently, continuum approximations are inapplicable to the physical processes of the forming section of the paper machine, except possibly in some parts of the headbox. Even though space averaged physical fields, such as solids concentration, stress tensor field, etcetera, can be defined in a consistent way, these fields are insufficient for formulating scalable constitutive relations for the mechanisms of forming, including suspension flow and dewatering. Hence, in order to model forming, it is a necessary to take the finite geometry of the fibres into account. The shortcomings of continuum models, and benefits of particle-level models, for dewatering will be further elaborated upon in later Sec. 4.4.1.
We proceed to make Reynolds number estimates to determine in which flow regimes the headbox and former operate. As usual, the Reynolds number is defined by

\[ \text{Re} = \frac{\rho u h}{\eta}, \]  

where \( \rho \) is the density and \( \eta \) is the viscosity of the fluid medium, \( u \) is the typical velocity of the flow, and \( h \) is the typical length scale of the flow. From the nozzle tip of the headbox to the drainage zone, we have \( h \sim 1 \text{ cm} \)—the thickness of the jet. The fluid medium is water. The density of the suspension thus becomes almost the same as water, whereas the viscosity is slightly higher than that of water due to the presence of fibres. For an order of magnitude estimate, however, it is sufficient to use the density and viscosity of pure water. It remains to estimate the typical velocity \( u \) of the flow in order to calculate the Reynolds number. It must be kept in mind that \( u \) should be taken as the flow velocity relative to the boundaries with which the fluid interacts. Otherwise, \( u \) would depend on the choice of reference frame.

The jet speed of modern paper machines is about \( u_j = 30 \text{ m/s} \), which is also the typical velocity at the headbox nozzle exit. Since no boundary layer is present in the free jet, the speed differences within the jet are due to turbulence and the flow structures created by the vanes. Turbulence caused by the boundary layer in the contraction is in the order of \( \frac{1}{40} u_j \) [48], while turbulence reminiscent from the tube bank up-stream is generally smaller in magnitude [49]. The amplitude of the secondary flow created by the vanes is in the order of \( \frac{1}{100} u_{\text{tip}} \) just after the tips of the vanes [48], where \( u_{\text{tip}} \) is the mean flow velocity at that point. All things considered, we estimate the magnitude of the secondary flows to be in the order of \( \frac{1}{30} u_j \), mainly due to the flow induced by the vanes. This value is of course heavily dependent on the headbox geometry and the properties of the vanes. During drainage, the typical speed differences in the fluid is equal to the difference between jet and wire speed, which is 1–2 % of the machine speed, and amounts to approximately \( 0.5 \text{ m/s} \). From these estimates it is possible to compute the Reynolds number during each phase of forming. The results have been compiled in Table 1. As is clearly seen, the flow is in the turbulent regime throughout the forming section.

We must distinguish between the flow regime for the large scale flow of the suspension, and the flow regime of the fibre–fluid interactions. The latter is characterised by the so called particle Reynolds number, defined as \( \text{Re}_p = \rho D u_p / \eta \). Here, \( D \) is the fibre diameter, and \( u_p \) is the cross-flow experienced by the fibres. A freely moving fibre will not be subjected to any cross-flow in a linear velocity gradient. A flow structure of size \( d \) and velocity \( u \), however, creates a nonlinear flow gradient, whose second derivative is in the order of \( u/d^2 \). Consequently, a freely moving fibre will be subjected to a cross-flow in the order of \( u_p \sim u L^2/d^2 \), which correspond to the velocity variations along the length of the fibre. If we assume that the size of the flow structures is about the same as the thickness of the jet, then in our case \( u_p \approx \frac{1}{10} u \).

Large wood fibres, which represent the worst case, have the dimensions \( L \approx 3 \text{ mm} \) and \( D \approx 30 \mu \text{m} \). The corresponding particle Reynolds numbers can be found in Table 1. The fibre–fluid interactions are in the non-creeping flow regime, but they are not turbulent.

Wood fibres tend to swell when submerged in water. Therefore, most of the particle mass in the wet state is due to the fluid contained within the fibre, and the density
Table 1. Estimates of the Reynolds number $Re$, the particle Reynolds number $Re_p$, and the Stokes number $St$, in the headbox, the jet, and the drainage zone of the forming section of a modern paper machine. The typical flow velocity magnitude $u$ of the jet refers to secondary flows, while the flow velocity magnitude at drainage was taken as the jet-to-wire speed difference. The large scale flow is in the turbulent flow regime throughout the process, while fibre–fluid interactions remain in the non-creeping regime.

<table>
<thead>
<tr>
<th>Position</th>
<th>$u$ [m/s]</th>
<th>$Re$ [-] (flow)</th>
<th>Regime (flow)</th>
<th>$Re_p$ [-] (interactions)</th>
<th>Regime (interactions)</th>
<th>$St$ [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Headbox</td>
<td>30</td>
<td>300,000</td>
<td>turbulent</td>
<td>100</td>
<td>non-creeping</td>
<td>0.8</td>
</tr>
<tr>
<td>Jet</td>
<td>1</td>
<td>10,000</td>
<td>turbulent</td>
<td>3</td>
<td>non-creeping</td>
<td>0.2</td>
</tr>
<tr>
<td>Drainage</td>
<td>0.5</td>
<td>5,000</td>
<td>turbulent</td>
<td>1</td>
<td>non-creeping</td>
<td>0.1</td>
</tr>
</tbody>
</table>

of the wet fibre becomes almost the same as the fluid density. It can be shown that, for an infinite circular cylinder with the same density as the suspending medium, the Stokes number in cross-flow is in the order of $St \sim C_D^{-1} D$, where $C_D$ is the drag coefficient for circular cylinders. The relation between $C_D$ and $Re_p$ is given in reference [50]. Table 1 lists the Stokes numbers relevant for the forming section. Since $St$ is close to unity, particle inertia is non-negligible.

The fibre coarseness, that is the fibre mass per unit length, is typically $\omega_f \in [0.15, 0.33] \cdot 10^{-6}$ kg/m for softwood [51], and $\omega_f \in [0.08, 0.25] \cdot 10^{-6}$ kg/m for hardwood [52]. From the concentration by mass $c_m$ of fibres, the length density $n_L = c_m/\omega_f$ can be calculated. The dimensionless concentration becomes $n_L^3 = c_m L^2/\omega_f$. To estimate it, we take the mass weighted mean fibre length for softwood to be $L = 2$ mm (see Table 1 of Paper III), and for hardwood $L = 1$ mm [53]. With $c_m \in [1, 10]$ g/l, we obtain $n_L^3 \in [4, 270]$. Further, if the fibre aspect ratio is approximate $r_p = 50$, we have $n_L^2 D \in [0.08, 5.4]$. The estimates reveal that the suspension is semidilute or concentrated; it is never dilute.

In the area of papermaking, an alternative terminology for the characterisation of concentration regimes is often encountered: Kerekes et al. [54] introduced the crowding factor $N_c$, defined as the average number of fibres within the rotational sphere of a single fibre. In a later communication, Kerekes and Schell [55] gave an approximate expression for the crowding factor of suspensions of swollen pulp fibres:

$$N_c \approx \frac{c_m L^2}{2 \omega_f}.$$  (1.3)

We note that $N_c \approx \frac{1}{2} n L^3$. The lowest concentration at which a fibre network can support load is $N_c \approx 16$ [56], and at $N_c \approx 60$, the fibres form a continuous network if they are well dispersed [57]. Typical values are $20 < N_c < 60$ for headbox concentrations [57]. In this regime, the suspension rheology of wood fibre suspensions depends heavily on the inter-fibre contact forces [54], including normal and frictional forces. However, according to Mason [58], colloidal forces are of secondary importance under the flow conditions in paper machines.

In summary, the paper sheet is formed in the semidilute or the concentrated regime. The suspending medium is primarily water, which behaves as a Newtonian fluid. The flow is in the non-creeping to the turbulent regime. Fibre–fluid in-
teractions are typically in the non-creeping flow regime. Fibres of various shapes interact with boundaries, with each other, and with the fluid medium. Due to these interactions, fibres may bend, and must consequently possess a finite stiffness. Also, particle inertia is non-negligible.

It can be seen immediately that none of the existing fibre suspension models, presented in Sec. 1.2.2, is suitable for solving the task at hand. Therefore, it is necessary to develop a new fibre suspension model, which takes into account all phenomena that may affect the development of the fibre configuration in the paper machine, and in the final paper. The new model must have an accuracy comparable to that of direct numerical simulations, while at the same time exhibit a computational efficiency exceeding that of previous models by orders of magnitude. In addition, the validity range of the model must include the operating conditions of the forming section of the paper machine.

1.4. System design

In addition to the issues concerning the physics of paper forming, it is of interest to discuss the simulation of forming from a software engineering point of view. A paper machine can be divided into serially arranged sections, each one corresponding to a unit process of papermaking. The system design used in this work adapts to this module-based design of the machine by allowing each unit processes model to be implemented as one or more stand-alone computer programs, whose inputs and output are files.

The input of a unit process model, or an attached end-use process model, is one file describing the raw material, stock or paper, and one file containing process parameters. The output is a new description of the raw material/paper that will be used as input to the subsequent unit process model. The arrangement of modules is illustrated in Fig. 4.

The design is intentionally kept open. Once the file formats in between unit processes has been specified, any developer can implement replacement unit process models in any programming language, or implement new tools for analysis of the paper descriptions.

Parameter files and paper description files have text based formats so that they can be read and edited in a common text editor. Since paper descriptions may be large, compressed text files\(^2\) are also allowed as input to, and can be requested as output from, the modules of the system.

It is an extensive task to develop particle-level models for all unit processes of papermaking. To study printing of uncoated grades, it is necessary to simulate at least forming, wet pressing, drying, calendering and the printing itself.

No work has been published previously, which seriously attempts to model the process of forming at a particle level, although Switzer et al. [42] simulated handsheet forming under Stokes flow conditions for the purpose of subjecting the resulting network structure to simulated mechanical testing. However, this work is aimed at providing a new particle-level model of paper forming, which embraces the full complexity of the problem.

\(^2\)The LZ77 algorithm and the file format of gzip are used for compression. Documentation can be found at http://www.gzip.org/.
Figure 4. Schematic overview of the proposed system for papermaking process simulation. The simulation software for each unit process is implemented in separate stand-alone executables. The modules communicate through files.
During wet pressing, calendering and printing the paper sheet is pressed by nips. Having this in common, it should be possible to simulate the processes using similar mechanical models, with some differences due to different moisture contents, and obviously with the addition of an ink transfer model in the case of printing. Models have been proposed by Provatas and Uesaka [59], and Drolet and Uesaka [60] for simulating the consolidation of fibre networks in a nip. A particle-level model has also been proposed for the application of coating [61]. These existing models may be included in the aforementioned system after some modifications. To the author’s knowledge, no particle-level model for the effects of drying on the paper microstructures has been proposed in the literature.

1.5. Outline of thesis

The main purpose of this work is to eventually make predictions of the end-use performance of papers (e.g. print quality development) based on the raw material properties and process parameters of papermaking. From the discussion above it is clear that this problem requires the modelling of all unit processes of papermaking at a particle level. As a first step, this thesis addresses the simulation of the forming unit process. A method for particle-level simulation of fibre suspensions is proposed in Sec. 2. Then, in Sec. 1.2, the results of the new fibre suspension model are compared with experimental data from the literature in the cases of isolated fibre motion and sheared fibre suspensions. A method for simulating forming in a roll–blade former (see Fig. 3) is presented in Sec. 4, and simulation results are discussed and compared with experimental data from the literature, and existing theoretical models. Finally, conclusions regarding the physics of fibre suspensions in general, as well as the physics of forming, are stated in Sec. 5.
2. THEORY

An Eulerian formulation, the vorticity–vector potential form of the incompressible Navier–Stokes equations, is used for the fluid, while fibres are modelled as discrete objects interacting with each other through contact forces, and with the fluid through drag forces. The two-way coupling between phases is taken into account by enforcing momentum conservation. A detailed description of the theory can be found in Paper I.

2.1. Fluid model

According to the problem formulation of Sec. 1.3, the suspension flow is in the turbulent regime. This implies that the flow model must be three-dimensional, because two- and three-dimensional turbulent flows manifest completely different dynamics. We will assume that the fluid is incompressible, with its density $\rho$ and viscosity $\eta$ constant over space and time. Under these conditions, the fluid motion is governed by the three-dimensional incompressible Navier–Stokes equations.

An efficient numerical method [62, 63, 64] for the solution of the Navier–Stokes equations on box-shaped domains, which is also relatively easy to formulate and implement, can be based on the vorticity–vector potential form of the Navier–Stokes equations:

\[
\begin{align*}
\frac{\partial \xi}{\partial t} &= \frac{\eta}{\rho} \Delta \xi - \nabla \times (\xi \times v) + \frac{1}{\rho} \nabla \times F^b \quad (2.1a) \\
\xi &= \nabla \times v \quad (2.1b) \\
v &= \nabla \times \Psi \quad (2.1c) \\
\Delta \Psi &= -\xi \quad (2.1d) \\
v(x, 0) &= v_0(x), \quad (2.1e)
\end{align*}
\]

where $\xi$ is the vorticity, $v$ is the velocity, $v_0$ is the initial velocity, $\Psi$ is the vector potential, and $F^b$ is the body force density. In this context, the body force density term includes non-potential forces; particularly, the particle pressure exerted on the fluid, as will be discussed in later Sec. 2.2.3.

The general boundary conditions, in the cases of fluid–solid, fluid–fluid and fluid–void interfaces, have been derived and proved by Hirasaki and Hellums [65]. The main difficulty with the vorticity–vector potential formulation is usually the boundary conditions. However, for plane boundaries, the conditions of fluid–void and fluid–solid interfaces take simple forms (see Paper I). Equations (2.1a)–(2.1e) are solved using a second-order finite-difference scheme [64]. For each iteration, the Poisson equation (2.1d) has to be solved. This is done using the Successive Over-Relaxation (SOR) method. The SOR method has the benefit of being able to exploit the fact that the solution varies very little between time steps. For the time discretisation, the Adams–Bashforth–3 method was implemented; higher-order methods somewhat reduce the difficulties with numerical instability, which are normally observed when solving the Navier–Stokes equations.

A subgrid turbulence model was included in order to allow for the calculation of large Reynolds number flows at a reasonable cell size of the spatial discretisa-
function. Consequently, Large Eddy Simulations (LES) were performed. According to Lesieur and Métai 
[66], the subgrid turbulence can be accounted for by adding the kinematic eddy viscosity

\[ \nu_s(x, t) = 0.105 C_K^{-3/2} \Delta x \sqrt{F_2(x, t)}, \tag{2.2} \]

to the true kinematic viscosity of the medium. Here, \( C_K \approx 1.4 \) is the Kolmogorov constant, \( \Delta x \) is the cell size of the spatial discretisation, and \( F_2 \) is the local second order velocity structure function defined by

\[ F_2(x, t) = \langle ||v(x, t) - v(x + \rho, t)||^2 \rangle_{||\rho|| = \Delta x}. \tag{2.3} \]

2.2. Fibre model

As indicated in Sec. 1.3, the fibre model must take fibre flexibility, normal and frictional forces in fibre–fibre interactions, and the two-way hydrodynamic interactions between the fibres and the fluid into account. The model must also include the dynamic drag forces, since the fibre–fluid interactions are not necessarily in the creeping flow regime.

In the work of Schmid et al. [40], a fibre model was proposed, which takes the fibre flexibility, mechanical fibre–fibre interactions and the actions of the fluid on the fibres into account for a prescribed fluid flow. It does not, however, consider particle inertia, hydrodynamic fibre–fibre interactions, the two-way coupling between phases or the non-creeping nature of the fibre–fluid interactions. Below, the model of Schmid et al. is further expanded to include the required features of the model.

Suspended fibres are modelled as chains of fibre segments, indexed \( i \in [1, N] \), of cylindrical cross-sections (see Fig. 5). Let \( r_i \) denote the position vector of the centroid of segment \( i \). The length of segment \( i \) is \( l_i \), and the unit vector pointing in the length direction of each segment is \( \hat{z}_i \).

To model fibre flexibility and fibres of various equilibrium shapes, each fibre segment is assigned its own local coordinate system. For each fibre segment \( i \), except for \( i = N \), there is an affine transformation mapping the local coordinate frame onto the coordinate frame of fibre segment \( i + 1 \). The set of transformations uniquely defines the equilibrium shape of the fibre. This enables the detection of deviations from the equilibrium shape and the calculation of the resulting bending and twist torques, which act to restore the equilibrium shape.
2.2.1. Equations of motion

Newton’s second law governs the motion of the fibre segments. Starting with Newton’s second law for translational motion, for fibre segment $i$, we have

\[ m_i \ddot{r}_i = F^h_i + F^w_i + \sum_j f_{ij} + X_{i+1} - X_i, \]  

(2.4)

where $m_i$ is the mass of the segment. On the right hand side of Eq. (2.4), $F^h_i$ represents the hydrodynamic force on segment $i$, $F^w_i$ is the body forces, such as weight, $\sum_j f_{ij}$ is the sum of particle and boundary interaction forces, and $X_i$ is the force exerted on segment $i - 1$ by segment $i$, which enforces the connectivity constraint.

Proceeding with Newton’s second law for rotational motion, we get

\[ \frac{\partial}{\partial t} (I_i \cdot \omega_i) = T^h_i + \sum_j t_{ij} + Y_{i+1} - Y_i + \]  

\[ \frac{l_i}{2} \dot{z}_i \times X_{i+1} + \left( -\frac{l_i}{2} \dot{z}_i \right) \times (-X_i). \]  

(2.5)

Here, $I_i$ is the time-dependent tensor of inertia of segment $i$, and $\omega_i$ is the angular velocity. Furthermore, $T^h_i$ denotes the hydrodynamic torque, $\sum_j t_{ij}$ is the sum of torques caused by particle and boundary interactions, $Y_i$ is the bending moment and torsional torque exerted on segment $i - 1$ by segment $i$, and the remaining terms represent the torque caused by connectivity forces.

To close the system, the connectivity constraint at the fibre segment joints must be expressed in terms of the velocity and the angular velocity. This is achieved by differentiating the connectivity constraint $r_i + l_i \dot{z}_i/2 = r_{i+1} - l_{i+1} \dot{z}_{i+1}/2$ with respect to time:

\[ \dot{r}_i - \dot{r}_{i+1} + \frac{l_i}{2} \omega_i \times \dot{z}_i + \frac{l_{i+1}}{2} \omega_{i+1} \times \dot{z}_{i+1} = 0. \]  

(2.6)

Now, equations (2.4)–(2.6) form the equations of motion for one fibre. They can be solved directly for segment velocity and angular velocity provided that expressions for $F^h_i$, $f_{ij}$, $X_i$, $T^h_i$, $t_{ij}$ and $Y_i$ can be found.

2.2.2. Fibre–fibre interactions

The interaction force $f_{ij}$ includes normal, frictional and lubrication forces. By the latter, we mean hydrodynamic forces acting between fibres on sub-grid scales. As suggested by Yamamoto and Matsuoka [37] and Schmid et al. [40], the normal force is taken as a repulsive force, which increases exponentially with fibre surface overlap, to prevent fibres from passing through each other. The frictional force is approximated from the normal force and the relative velocity between segments. A first-order approximation of the lubrication forces was used, based on the theoretical work of Yamane et al. [67] and Kromkamp et al. [68].
Figure 6. Fibre–fluid interactions. A fibre segment ➀ disturbs the velocity field of a region of fluid ➁, by first-order approximation, at length scales equal to or smaller than the fibre segment length. The cell size of the three-dimensional spatial discretisation ➂ of the fluid flow solver is equal to the segment length, in order to capture the flow of larger length scales.

2.2.3. Fibre–fluid interactions

The hydrodynamic drag force $F_h^i$ and torque $T_h^i$ on segment $i$, due to the flow velocity $v$, can be expressed as

\begin{align}
F_h^i &= A_h^i \cdot (v(r_i) - \dot{r}_i) \\
T_h^i &= C_h^i \cdot (\Omega(r_i) - \omega_i) + H_h^i : \dot{\gamma}(r_i),
\end{align}

where $A_h^i, C_h^i$ and $H_h^i$ are hydrodynamic resistance tensors, $\Omega = \frac{1}{2}\nabla \times v$ is the angular velocity of the fluid, and $\dot{\gamma} = \frac{1}{2}(\nabla v + (\nabla v)^T)$ is the rate of strain tensor. Consult Paper I for a detailed description of the hydrodynamic resistance tensors.

Equations (2.7a) and (2.7b) take into account the disturbance of the fluid created by the segment, and thereby the features of the flow at length scales smaller than, or comparable to, the fibre segment length. It is thus the responsibility of the flow model to capture length scales larger than the fibre segment length. Therefore, the cell size $\Delta x$ of the spatial discretisation of the flow solver was taken as the fibre segment length (see Fig. 6).

By virtue of the conservation of momentum, the impulse $F_h^i \Delta t$ and impulse moment $T_h^i \Delta t$, delivered to fibre segment $i$ by the fluid, must be compensated for by an impulse $-F_h^i \Delta t$ and an impulse moment $-T_h^i \Delta t$ acting on the fluid. Since the spatial discretisation does not resolve the details of the fibre surface, it is sufficient to represent the hydrodynamic interactions with a grid of point forces. These point forces are mapped onto a discrete representation of a force density field $F^p$, being the particle pressure exerted on the fluid. Momentum is conserved, simply by adding $F^p$ to the body force density $F^b$ of Eq. (2.1a).
2.2.4. Numerical stability

An implicit scheme, where the velocities and the angular velocities of the fibre segments are treated as unknowns, is used for the time discretisation of Eqs. (2.4)–(2.6). Ideally, all interacting fibres should be compiled into a common set of equations, to attain as small a numerical error as possible. Schmid et al. [40] neglected lubrication interactions, and as a result, they were able to identify fibre flocs which had no interactions with the neighbouring fibres. The problem size was thus reduced. However, when hydrodynamic interactions are taken into account, the probability of finding clusters of fibres not interacting with their neighbours becomes small even at moderate concentrations. Therefore, in this work, the equations of motion are solved for one fibre at a time. This is accomplished by using previous time steps to approximate the position, velocity and angular velocity of neighbouring fibres (see Paper I).

When Schmid et al. [40] used Eqs. (2.4)–(2.6) for calculating the motion of mass-less fibres subjected to a prescribed flow field, they noted that, the stiffer the fibre, the smaller time step was needed to ensure stability of the calculations. Similarly, the smaller the fibre segment length, the smaller the time step. In Paper I, a stability analysis of fibre bending was made, which showed that the time discretisation of Eq. (2.5) will be stable only if

$$\Delta t \lesssim \frac{\eta l^4 (1 + Re_p)}{E Y I},$$

(2.8)

where $E Y$ is Young’s modulus, $I$ is the area moment of inertia, and $l$ is the fibre segment length. A similar relation can be found for the torsion of fibres. Indeed, the computational cost increases with increasing fibre stiffness and decreasing fibre segment length. Since papermaking fibres are quite stiff ($E Y I \in [10^{-12}, 10^{-10}]$ Nm$^2$, see Ref. [69]), it is not feasible to use Eqs. (2.4)–(2.6) for simulating papermaking conditions, unless the numerical stability is improved.

It is described in Paper I, how artificial damping can be introduced in the fibre segment joints. By virtue of the damping, $\Delta t$ can be increased without jeopardising numerical stability at a small cost of accuracy. It was shown that, for stiff fibres, artificial damping can increase the computational speed by orders of magnitude, while the relative error in the amplitude of the bending angle is contained within 1%.

2.3. Algorithm

Given the solvers for fluid motion and fibre motion, and a model for the interaction between phases, a scheme for calculating the dynamics of fibre suspensions can be formulated. It is assumed that the initial velocity field $v_0$ of the fluid flow and the initial configuration of fibres are given.

1. Set the field $F^p$, representing the particle force density acting on the fluid, to zero.

2. Detect and store all fibre segment interactions.

3. For each fibre,
3.1. Solve the equations of motion for $\dot{r}_i$ and $\omega_i$, $i \in [1, N]$.

3.2. Move the fibre segments one incremental step.

3.3. Calculate the hydrodynamic forces and map them onto $F^p$, with a reversed sign.

4. Perform a forward step in the flow calculations, with $F^p$ included in the body force density term $F^b$.

5. Return to step 1.
3. FIBRE SUSPENSION RHEOLOGY

The predictions of the proposed fibre model are compared with experiments from the literature for the case of isolated fibre motion in Paper I, and for multiple interacting fibres in Paper II and Paper IV. The main purpose of Papers I and II is to validate the model with well controlled experimental results. The ranges validated are $r_p \in [20, 833]$ for the fibre aspect ratio in the case of isolated fibres, and $nL^3 \in [0.5, 50]$ for the dimensionless concentration in the case of multiple interacting fibres. Paper IV aims to establish relations between the stress components of the suspension and the fibre properties and flow conditions.

3.1. Motion of isolated fibres

The motion of isolated, rigid fibres in creeping flow is quite well known by virtue of the theoretical work of Jeffery [14], who studied the motion of ellipsoids immersed in a viscous fluid. Ellipsoids tend to perform repeatable orbits, so called Jeffery orbits, in shear flow. It was later discovered by Bretherton [17] that the same orbital behaviour can be expected from almost any body of revolution, including circular cylinders. Cox [19] derived a semi-empirical expression for the orbit period of circular cylinders.

Simulations in the range $r_p \in [5, 150]$ (see Paper I), using the proposed fibre suspension model, predicted the period of Jeffery orbits with a maximum error of 3.4%, as compared to the semi-empirical prediction of Cox [19], which, in turn, has been fitted by Cox to the experimental data of Anczurowski and Mason [16]. When the two-way coupling was neglected, in a second series of numerical experiments, large errors in the orbit period were seen. These two observations indicate that the two-way coupling between phases is essential to the motion of isolated fibres, and that the proposed fibre suspension model accurately accounts for the mutual fibre–fluid interactions.

As the fibre flexibility is increased, fibres start to buckle [70, 71]. To investigate the mechanical aspects of the fibre model, simulations were compared with the experiments of Salinas and Pittman [72], who measured the amount of bending of flexible fibres in creeping shear flow. Reasonable agreement with experimental observations was found for small and moderate amounts of bending. For radii of curvature less than $30D$, simulation predictions deteriorated. The discrepancy may have arisen from ignored effects, such as variations in the shape of the fibres, nonlinear material behaviour, and variations in material properties along the length of the fibres. The deviations may also have arisen from discretisation errors.

Experiments concerned with quantitatively describing fibre motion are only available in the literature for fibres of small and moderate flexibility. As flexibility is further increased, the fibre motion becomes less ordered. Forgacs and Mason [71] observed five different regimes of motion for flexible fibres in creeping shear flow: rigid motion, springy motion, snakelike motion, coiled motion, and coiled motion with self-entanglement. The rigid, springy, and snakelike regimes of motion were observed in simulations with the proposed fibre suspension model (see Fig. 7). For originally straight fibres, no coiled motion could be observed in simulations. Forgacs and Mason provided several series of photographs taken of the orbiting fibres. In the
Figure 7. Five regimes of motion observed for flexible fibres in shear flow. The sequences of images are composed from snapshots of the simulations of (1) rigid motion, (2) springy motion, (3) snakelike motion, (4) coiled motion, and (5) coiled motion with self-entanglement. The coiled motion (4 and 5) was provoked by attributing a residual curvature to the fibres; no coiled motion could otherwise be observed in simulations.
case of snake rotation a straight fibre should straighten out completely once every half-revolution. The photographs of Forgacs and Mason, however, shows that that the fibre developed a residual curvature, whose radius was at most 1.5 mm, as measured directly from the photographs. Since bending is even greater during coiled motion, the residual curvature must also be greater. By attributing permanent deformations, corresponding to a radius of curvature of 0.7 mm, to a 4.0 mm long fibre, it was possible to provoke the coiled motion in simulations. Similarly, it was possible to provoke coiled motion with self-entanglement for 10.0 mm long elastomer fibres. Plastic deformations caused by bending may thus be responsible for the coiled motion of high flexibility fibres.

Stockie and Green [44], who modelled the motion of flexible fibres in shear flow in two dimensions, qualitatively predicted the rigid, springy and snakelike regimes of motion. They did not, however, observe the coiled motion due to its three-dimensional nature. In three dimensions, Schmid et al. [40] qualitatively predicted the rigid, springy and snakelike motion as well. Qi [47], simulated the rigid and springy fibre motion in three dimensions, taking the two-way coupling between phases into account. No previous numerical investigation have quantitatively predicted the amount of fibre bending, or qualitatively predicted the coiled regimes of motion.

From the simulations, it was concluded that the proposed model predicts the motion of isolated fibres in creeping flow for a wide range of fibre flexibilities and fibre aspect ratios. It is thus reasonable to assume that the model accurately captures the mechanical behaviour of fibres, as well as the two-way coupling between the fibres and the fluid phase. As for the case of finite particle Reynolds numbers, the validation was then focused on the fibre–fluid interactions. Cross-flow over a fixed rigid fibre was simulated, and the resulting hydrodynamic drag coefficient $C_D$ was compared with experimental data for cross-flow over a circular cylinder [50, 73], for a very wide range of Reynolds numbers $Re \in [10^{-3}, 10^4]$. This comparison is illustrated in Fig. 8. The maximum discrepancy is 42% at $Re = 5.4$. It is at these intermediate Reynolds numbers that the interference effects of the skin friction drag and the pressure drag are at their greatest. Neglecting these interference effects, it can be said that the model roughly approximates the hydrodynamic drag forces at finite particle Reynolds numbers.

### 3.2. Sheared fibre suspensions

The rheology of sheared fibre suspensions were investigated in Papers II and IV. In Paper II, the fibre suspension model was validated for the case of multiple interacting fibres. No additional theory was required for extending the model to this case, as compared to isolated fibres. Paper IV was concerned with explaining the mechanisms behind the development of the fibre suspension stresses. Both Papers II and IV were restricted to straight, rigid fibres, for which there are many experimental studies available for comparison in the literature, particularly for simple shear flow [27, 28, 30, 74, 75, 76].

Simulations were conducted for monodispersed suspensions of straight, rigid, non-Brownian fibres in creeping shear flow, with a Newtonian fluid medium. Three independent input parameters were varied—the fibre aspect ratio $r_p$, the dimension-
less concentration $nL^3$, and the coefficient of inter-particle friction $\mu_l$—in order to investigate their effects on suspension rheology. A box-shaped computational domain was used, where two opposing boundaries were assigned moving wall boundary conditions, while the rest of the boundaries were assigned periodic boundary conditions. Simple shear flow was maintained by imposing lateral motion to the solid wall boundaries. The initial fibre configuration was drawn from an isotropic fibre orientation distribution and a uniform spatial distribution.

During the transient, the rheological properties vary periodically with decreasing amplitude, as was observed in experiments by Ivanov and co-workers [77, 78] for the apparent viscosity. After some time, as more shear is applied, the rheological properties approach asymptotic values. This work is concerned with these asymptotic values, henceforth referred to as the steady state values. The steady state values were taken as the time average over two complete Jeffery orbit periods, after the decay of the transient phenomena. Similarly, steady state orientation distributions were obtained by time-averaging after the transient.

### 3.2.1. Fibre orientation distribution

The orientation vector $\hat{p}$, pointing along the length of a straight fibre, has two degrees of freedom, and can thus be represented by two parameters. Traditionally, the azimuthal angle $\phi \in [0, 2\pi)$ with the $x_3$ axis as the polar axis, and the orbit constant $C$, defined by [21, 79]

$$C = \frac{1}{r_c[p_3]} \sqrt{r_c^2 \hat{p}_2^2 + \hat{p}_1^2},$$  

(3.1)
Figure 9. Definition of coordinate system and orientation angles $\phi$ and $\theta$. The orientation vector $\hat{p}$ of a fibre traces a path on the unit sphere, characterised by the orbit constant $C$, when the isolated fibre is subjected to shear flow.

are used for this purpose. Here, $r_e$ is the equivalent aspect ratio for circular cylinders, for which a semi-empirical expression is available \[19\]. Figure 9 illustrates the geometrical definitions. Isolated fibres following Jeffery orbits have a fixed orbit constant. If there are hydrodynamic or mechanical interactions with other fibres or boundaries, or if the effects of fibre bending, fibre inertia, or the particle Reynolds number are non-negligible, the orbit constant will vary over time.

The fibre orientation distribution can be described by the azimuthal angle distribution $p(\phi)$ and the orbit constant distribution $p(C)$. For convenience, the corresponding distributions $p^+(\phi) = p(\phi) + p(\phi + \pi), \phi \in [0, \pi)$ and $p(C_b), C_b \in [0, 1]$ with $C_b = C/(1 + C)$, are studied.

The experimental investigation of Stover et al. [27] revealed that the $\phi$ distribution in the semidilute and the concentrated regimes deviates little from the time averaged $\phi$ distribution of isolated fibres. The simulations confirmed this result; the $\phi$ distribution of an isolated fibre from Jeffery [14] is compared with that of a sample simulation in Fig. 10. Fibres stay, roughly, in the sheared plane ($x_1x_2$ plane) even in the semidilute and the concentrated regimes.

Stover et al. [27] also measured the $C_b$ distribution in the semidilute and the concentrated regime. A comparison with simulation results is given in Fig. 11. It can be seen that the model predicts very well the $C_b$ distribution. It was also shown in Paper II that the mean value of the $C_b$ distribution compared well with experimental results from the literature [27, 28]. It was thus concluded that the proposed fibre suspension model accurately predicts the fibre orientation distribution in the semidilute regime, and the transition region between the semidilute and the concentrated regime.
represents the theoretical prediction of Jeffery (see Ref. [14]) for isolated fibres. The bars represent the computed distribution for the dimensionless concentration \( \phi \pi [-] \) and simulation results. Hydrodynamic and mechanical interactions are responsible for the discrepancy between Jeffery’s prediction and simulation results.

Figure 10. The \( \phi \) distribution of fibres with aspect ratio \( r_p = 20.3 \) in shear flow. The solid line represents the theoretical prediction of Jeffery (see Ref. [14]) for isolated fibres. The bars represent the computed distribution for the dimensionless concentration \( nL^3 = 2 \).

Figure 11. The \( C_b \) distribution measured by Stover et al. (see Ref. [27]), for \( r_p = 31.9 \) and the volume concentration \( c = 0.035 \) (triangles), and the \( C_b \) distribution obtained from simulations with the same parameters (solid line).
3.2.2. Spatial distribution of fibres

The fibres of flowing fibre suspensions tend to flocculate at certain conditions. It has been verified experimentally that flocculated suspensions exhibit markedly different rheological behaviour than well dispersed suspensions. For instance, flocculated suspensions are shear rate dependent, and the apparent shear viscosity of a flocculated suspension is always greater than that of the corresponding dispersed suspensions of the same concentration [74]. Therefore, before we proceed to investigate the stress tensor components of fibre suspensions, it is essential to gain some insight in the development of a flocculated state.

In Paper IV, the intensity of flocculation was defined as

\[ P_L = \frac{s^2_L(n L^3)}{n L^3}, \]  

(3.2)

where \( s^2_L(\ldots) \) denotes the variance of a local quantity in many volumes \( L^3 \). Note that \( n \) is the number concentration of fibre centroids. The number of fibres in the volume elements would be Poisson distributed for randomly uniform fibre configurations. Moreover, it is a property of the Poisson distribution that its variance is equal to its mean. Therefore, we have \( P_L = 1 \) for randomly uniform fibre configurations. In a flocculated suspension \( P_L > 1 \), while \( P_L < 1 \) indicates that dispersive mechanisms are present. The definition of \( P_L \) is analogous with the formation number for two-dimensional fibre networks, which is defined as the ratio between the measured variance of local grammage and the variance expected for a randomly uniform network [80].

The simulations showed that the time averaged intensity of flocculation \( \bar{P}_L \) was approximately equal to 1 when \( n L^2 D \lesssim 0.5 \). At greater concentrations, \( \bar{P}_L \) attained values significantly greater than 1, indicating flocculation. It was also observed that \( \bar{P}_L \) increased with inter-fibre friction.

When flocs develop, the floc size, rather than the fibre length, becomes the maximum inherent length scale of the suspension. In this work, the size of the simulation box was of the same order as the floc size. The accuracy of the simulation predictions may thus be compromised as soon as the suspension reaches a flocculated state. Therefore, it should be emphasised that reasonable quantitative predictions of rheological properties are only to be expected when \( n L^2 D \lesssim 0.5 \).

3.2.3. Fibre contribution to suspension stresses

Batchelor [24] asserts that the stress tensor of suspensions of freely moving, straight, rigid fibres is on the form

\[ \sigma = \eta_{fib} \left( \langle \hat{p} \hat{p} \hat{p} \hat{p} \rangle - \frac{1}{3} \delta \langle \hat{p} \hat{p} \rangle \right) : \dot{\gamma} + 2\eta \dot{\gamma} \]  

(3.3)

where \( \delta \) is the unit tensor, \( \dot{\gamma} \) is the rate of strain tensor, \( \eta \) is the viscosity of the medium, and \( \eta_{fib} \), the “fibre viscosity”, is a function of concentration, orientation distribution, and fibre geometry. For semidilute suspensions, which are the main concern in this work, Shaqfeh and Fredrickson [26] derived the expression

\[ \eta_{fib} = \frac{\pi n L^3 \eta}{3 (A - \ln c + \ln (-\ln c))} \]  

(3.4)
where \( c \) is the volume concentration of fibres, and \( A \) is a function of the orientation distribution. The specific viscosity \( \eta_{sp} \), the dimensionless first normal stress difference \( N_1 \), and the dimensionless second normal stress difference \( N_2 \) follow from Eq. (3.3):

\[
\eta_{sp} = \frac{\eta_{app}}{\eta} - 1 = \frac{\eta_{fib}}{\eta} \left< \hat{p}_1^2 \hat{p}_2^2 \right>,
\]

\[
N_1 = \frac{1}{\eta G} (\sigma_{11} - \sigma_{22}) = \frac{\eta_{fib}}{\eta} \left< \left( \hat{p}_1^3 \hat{p}_2^3 - \hat{p}_1 \hat{p}_2^2 \hat{p}_3^2 \right) \right>,
\]

\[
N_2 = \frac{1}{\eta G} (\sigma_{22} - \sigma_{33}) = \frac{\eta_{fib}}{\eta} \left< \left( \hat{p}_1 \hat{p}_2^3 - \hat{p}_1 \hat{p}_2 \hat{p}_3^2 \right) \right>.
\]

Here, \( G \) is the shear rate. See Paper IV for further details.

The above theoretical model may be compared with the fibre suspension model that renders the full time-dependent state description of the suspension, including details of the flow, and the distribution of hydrodynamic forces along the lengths of the fibres. It is thus possible to directly compute the average traction on any section of the suspension. By computing the traction exerted on three orthogonally oriented planes, the stress tensor may be computed directly from its definition. The viscosity and first/second normal stress differences, in turn, can be computed directly from the stress tensor components, as usual, using their respective definitions.

That is, with the proposed model, the stress tensor can be computed without having to rely on the various assumptions and estimates associated with the slender body theory \([24, 25, 26]\), which is otherwise applied when calculating the rheological properties of a fibre suspension from some given configuration of fibres.

Two fundamentally different methods of computing the rheological properties from the state of the suspension have been defined above. Henceforth, rheological properties obtained using Eqs. (3.3) and (3.4) will be referred to as theoretical, while rheological properties obtained from direct computation of the stresses will be referred to as computed. As an example, the theoretical and the computed dimensionless first normal stress differences are compared in Fig. 12, for the parameter set \( \{ r_p = 30; n L^3 = 3.8; \mu_s = 0.0 \} \). In this case, the two predictions are almost identical. Also note the typical behaviour of all rheological properties, with a transient and an asymptotic steady state.

If fibres interact solely through hydrodynamic interactions, the steady state fibre orientation distribution would necessarily be symmetric with respect to the \( x_1x_3 \) plane \([81, 82]\). Then, according to Eq. (3.5b), the steady state first normal stress difference \( N_1 \) would vanish. In the simulations, both theoretical and computed results indicated that \( N_1 \) was finite and positive throughout the investigated range of concentrations \( n L^3 \in [1, r_p] \), with zero inter-particle friction. By inference, non-hydrodynamic particle interactions, e.g. contact forces in the normal direction, must have been responsible for this phenomenon.

In Paper IV, it was shown how the fibres of a non-concentrated suspension can be separated into two disjoint fractions: One consisting of fibres recently involved in a mechanical contact interaction with some other fibre, and the complementary “undisturbed” fibre fraction. The orientation distribution of the undisturbed fraction was assumed to be symmetric, so that it does not contribute to \( N_1 \). An approximation of the sizes of the respective fractions were derived analytically, and it was
shown by numerical experiments that the size of the disturbed fraction was the key parameter controlling the development of \( \overline{N}_1 \). A semi-empirical relation was thus obtained for the dimensionless first normal stress difference:

\[
\overline{N}_1 = (g_0 + g_1 nL^2 D \mu_t) \cdot nL^3 \cdot \frac{\ln r_e}{r_e r_p} \cdot \frac{\eta_{fib}}{\eta},
\]

where \( g_0 \approx 0.18 \) and \( g_1 \approx 0.54 \).

As opposed to the results for \( N_{1f} \), theoretical results deviated from computed results for \( N_2 \). This indicates that effects of mechanical contacts dominate over hydrodynamic effects. Consequently, Eq. (3.5c) cannot be used to predict the steady state second normal stress difference \( \overline{N}_2 \). Similarly to the case of \( \overline{N}_1 \), the most important parameter controlling the development of \( \overline{N}_2 \) was the size of the disturbed fibre fraction. A coarse estimate of \( \overline{N}_2 \) was obtained for the case of zero inter-particle friction:

\[
\overline{N}_2 = K \cdot nL^3 \cdot \frac{\ln r_e}{r_e r_p} \cdot \frac{\eta_{fib}}{\eta},
\]

where \( K \approx -0.0066 \).

Simulation results for the specific viscosity at \( \mu_t = 0.20 \) are compared with the experimental data of Blakeney [74], Bibbo [75], and Petrich et al. [28] in Fig. 13. It was observed in Paper IV that, in cases where the suspension was well-dispersed, the specific viscosity fell on a master curve:

\[
\eta_{sp} = \bar{\eta}_0 \left( \frac{1}{r_e} \cdot \frac{\eta_{fib}}{\eta} \right)^{\lambda},
\]

where \( \bar{\eta}_0 \approx 0.474 \) and \( \lambda \approx 1.22 \). Deviations from this master curve can be explained by flocculation: Petrich and co-workers added a dispersive agent to their suspensions in order to prevent flocculation. Therefore, their results follow the master curve for all investigated concentrations. The present simulations, however, deviates from the master curve at the greatest concentration, because a flocculated state developed at that point.
Figure 13. The steady state specific viscosity $\eta_{sp}$ plotted as a function of $\eta_{fib}/(\eta_r e)$. The $\times$es denote simulation results, while the open symbols denote experimental data in the literature: Blakeney, $r_p \approx 20$, squares, see Ref. [74]; Bibbo, $r_p \in [17, 33]$, triangles, see Ref. [75]; Petrich, $r_p = 50$, circles, see Ref. [28]. The solid line indicates an empirical power law found for well-dispersed suspensions.
4. SIMULATION OF FORMING

Given the general fibre suspension model described in the previous chapter 2, and the positive results of its validation in chapter 3, we are equipped to take on the task of formulating a particle-level model for the forming section. For this purpose, material from Paper III is reviewed in this chapter. Also, an additional discussion is presented, which addresses the development of the dewatering pressure in simulations.

4.1. Stock

The raw material input to the forming section is the stock. In this forming simulation model, it is described statistically. Fibres have varying length, diameter, wall thickness, stiffness, curvature, and so on. To correctly model fibre properties and capture the interdependence between these properties correctly, the fibres should be described by an $n$-dimensional probability distribution of general shape. However, information about such a distribution is difficult to obtain experimentally.

In this work (see Paper III), the PulpEye$^\text{TM}$ fibre analyser was used to obtain the three-dimensional probability distribution of fibre length, diameter, and curl index. The remaining fibre properties were regarded as independent, and their typical distributions were obtained from the literature. Three-dimensional model fibres were then created from data drawn from the distributions. A comparison between an original kraft fibre furnish and the corresponding computer generated furnish is presented in Fig. 14. In order to maintain reasonable execution time, all particles smaller than 200 $\mu m$ were removed from the model suspension due to their vast number. Fillers and fines have thus not been included in the model at this point.

In the simulations of Paper III, the crowding factor was $N_c \approx 45$, obtained by inserting the mass weighted averaged fibre properties into Eq. (1.3). Considering the fact that the lowest concentration at which a fibre network can support load is $N_c \approx 16$ [56], the mobility of the fibres is severely restricted at $N_c \approx 45$.

4.2. Subprocesses of forming

In much the same way as the papermaking process can be broken down into serially arranged unit processes (see Sec. 1.1.2), each unit process can be broken down into subprocesses. However, in the case of forming, this is not exactly true; particle-laden fluid that has been drained during forming is reintroduced up-stream of the headbox. Therefore, it is not possible to divide the entire forming section into serially arranged subprocesses due to this recirculation of the so-called white water. Instead, we choose to model the forming section from a point after the white water inlet, and thus ignore the recirculation.

With this approach it is possible to identify at least three subprocesses of forming, associated with the headbox, the jet, and the drainage zone, respectively (see Fig. 15). In these three zones, there are fundamental differences in the flow conditions (see Sec. 1.3) and in the boundary conditions imposed on the suspension.
Figure 14. Image (a) shows the photograph of a kraft, softwood fibre furnish analysed by the PulpEye™ fibre analyser. For comparison, image (b) shows a population of model fibres drawn from the corresponding statistical description of the furnish. All particles smaller than 200 µm have been removed from the distribution for the reason of computational cost. The heights of the images are 5.1 mm.

Figure 15. Schematic overview of the subprocesses of forming. Of these, the free jet and the drainage are subject to particle-level simulations in this work. The broken line denotes the white water recirculation, which is not taken into account by the present model.
4.2.1. Headbox

In the headbox, the typical size of the flow domain is several times greater than the jet thickness, and the Reynolds number is at least an order of magnitude greater than in the jet—troublesome facts that make numerical simulation of the particle-level dynamics prohibitively demanding in this region with the present model. Therefore, it is not the focus of this work to model headbox phenomena. Instead we employ an existing model, proposed by Olson and co-workers [8, 9], to create the initial condition for the jet.

The main purpose of the headbox is to produce a wide, thin jet of fibre suspension. The initial condition of the jet consists of the instantaneous velocity field of the flow at the nozzle tip of the headbox, and the details of the geometry of the fibre phase. Three-dimensional model fibres are created, using material and morphological data drawn from the multi-dimensional probability distribution of the fibre properties. The fibre positions are drawn from a randomly uniform spatial distribution on a box-shaped region $\Omega$, and their orientations are drawn from an orientation distribution approximating the state at the nozzle tip [9]. The initial velocity field $v_0(x)$ at the nozzle tip was taken to be a fully developed turbulent flow field.

4.2.2. Jet

Given the initial condition, simulating the jet is simply a matter of integrating the fibre and fluid motion using the fibre suspension model proposed in previous chapter 2. The simulation box $\Omega$ and a Cartesian coordinate system move with the speed of the jet so that the mean velocity of the fluid becomes zero with respect to the coordinate frame. Free-slip boundary conditions were used at the fluid–air boundaries, and periodic conditions were enforced on the other boundary planes. These boundary conditions are further discussed in Sec. 4.3.

When the motion of the fluid–particle system has been integrated for the duration of transport of fluid from the nozzle tip to the impingement point on the wires, the initial condition for the simulation of drainage has been obtained.

4.2.3. Drainage

Drainage is modelled inside a box-shaped computational domain $\Omega$, which moves with the speed of the wires. Two model forming fabrics are introduced in $\Omega$. The yarns of the model fabrics interact with the fibres of the suspensions through the same contact mechanics model as for inter-fibre contacts. The interactions between the wire yarns and the fluid are captured using the immersed boundary method (see Paper III and Ref. [43]). The two wires are originally separated by a distance equal to the jet thickness. Drainage is simulated by moving the model wires toward each other. The fibres still trapped between the wires at the end of the simulation constitutes the final paper web.

Figure 16 illustrates the simulation geometry. The model suspension flowing between and outside the wires is allowed to have a non-zero average speed in the $x_1$ direction to account for the jet-to-wire speed difference $\Delta u_{jw}$. Since only a small volume of suspension is considered in the particle-level simulations, it is not possible to predict global features of the flow, such as the development of the mix-to-wire
should be pointed out that the amplitude of the pressure variations (was taken from the 500 m/min pilot machine measurements of Bando time-dependent, indicating wire motion in the x-direction. The rates of these parameters also represent drainage rates from the top and bottom wires. They should be given by direct experimental measurements, or can be numerically estimated with the present model from the experimental data of the pressure difference between the core of the suspension and the atmosphere. Unfortunately, direct measurements are currently unavailable in the literature. In addition, the pressure difference data, corresponding to the pulp stock used in this simulations\(^1\), is not available either. Therefore, in Paper III, the rate of dewatering on the roll was estimated using the power law model of Ingemarsson [83].

In the simulations of Paper III, the pressure variations \(p_m(t)\) of the suspension was taken from the 500 m/min pilot machine measurements of Bando et al. [84]. It should be pointed out that the amplitude of the pressure variations \(p_m \approx 2.5 \text{kPa}\) over the blades in their trials was in the lower range; blade pressures exceeding 10 kPa were reported by Bergström et al. [85] for a comparable machine speed. The Bernoulli equation was used to obtain the mix-to-wire speed difference \(\Delta u_{mw}(t)\) variations from the experimentally obtained pressure variations \(p_m(t)\).

The simulations render fibre network structures (see Fig. 17), which can subsequently be analysed, or used as input in simulations of unit processes down-stream.

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\(^1\)Since fibrils, fines, and fillers have been removed from the model suspension in this work, the pressure difference between the core of the suspension and the atmosphere would be underestimated.
4.3. Limitations of the model

Even though the fibre suspension model was carefully validated in Paper I and Paper II through comparisons with experiments, its application to the forming process is associated with some simplifications. These must be carefully considered when simulation results are interpreted.

The initial fibre configuration and the initial flow field at the nozzle tip of the headbox are presently not accessible experimentally. Therefore, models have been used to generate the initial state (see Sec. 4.2.1). These models produce well-dispersed fibre configurations, and flocculation develops later, during the simulation of the jet (see Paper III). In reality, the suspension is flocculated to some degree already at the nozzle tip.

In the simulations, the fluid–air interfaces of the jet are restricted to plane surfaces. Therefore, some of the important effects of the free surface flow on the structure of the jet are neglected. In a real jet flow, the jet thickness increases near the impingement point, and the free surface flow in its vicinity becomes irregular. These phenomena are left for future work.

Throughout this work, it has been assumed that an equal amount of fluid is drained through the top and bottom fabric at each drainage element, so-called symmetric drainage. In reality, the drainage is always asymmetric to some degree, leading to two-sidedness of the paper structure. This missing feature of the simulations is not due to any shortcomings of the model, but to the availability of experimental data in the literature.

In mechanical pulps, a large fraction of the solids are smaller than the 200 µm lower particle size limit chosen for these simulations. Fines and fillers tend to accumulate in the white water recirculation (see Fig. 15). These small particles are known significantly affect the suspension rheology as well as the mechanisms of de-watering. The basic effects of these small particles can be included in the model.
Figure 18. The mechanical and chemical treatment of the pulp prior to paper forming result in fibrillation. The surfaces of the fibres rupture and thin bundles of fibrils (three of them indicated by arrows) extend from the fibre surface. The image shows fibres from a TMP pulp. (Courtesy of B. Nilsson, SCA R & D Centre AB)

framework, either by utilisation of modern supercomputers to allow for a reduction of the lower particle size limit, or by introduction of density fields for the respective small particle fractions.

The mechanical and chemical treatments, that pulps are subjected to in processes up-stream of the former, result in rupture of the fibre surface. Thin fragments of fibrous material called fibrils extend from the fibre surface (see Fig. 18). Fibrils affect the fibre–fluid interactions, and, since they may promote fibre entanglement, the nature of fibre interactions as well. If these interactions are quantified experimentally, the effects of fibrillation can be accounted in the proposed fibre suspension model.

The present fibre suspension model is not well-suited for predicting the mechanics of high concentration fibre networks. Therefore, the sheets are only compressed up to an average solids concentration of approximately 10%. At high concentrations, momentum is primarily transferred through the fibre–fibre contacts of the fibre network, and not through the fluid. Therefore, for modelling the final compression of the network, and further consolidation during wet pressing, another modelling framework is required, which more carefully considers the fibre network mechanics at high concentrations.

During the final phase of dewatering, the fibre network reaches a concentration of 5–10%. At this point, the fibres form a network, which may be strong enough not to yield subject to the pressure gradient induced by the last blade. Then, the mix no longer obeys the Bernoulli equation, which was used to compute the lateral flow of the fluid. We may thus expect that the magnitude of the mix-to-wire speed over the final blade is overestimated in the later section of forming.

Since the limitations of the current model have been reviewed, it may be appropriate at this point to discuss the limitations of the previous models proposed for forming [86, 87, 88, 89, 90]. In the previous models, the fibre suspension and wires are modelled as continua, with effective properties, such as viscosity, permeability, etc. Therefore, any microscopic phenomena taking place at the length scales of fi-
Figure 19. The sequence of images shows the evolution of the fibre network in a thin slice of the simulation box. The machine direction is to the right in the images. Image (a) was taken at the impingement point, image (b) after the roll, image (c) after three blades, and image (d) after all six blades. The height of each image is 6.25 mm in reality. Note that no appreciable through-thickness concentration gradient can be seen, except in the last image.

bre length and wire yarn diameter are automatically precluded. In other words, the models can not capture the essence of the problem (formation, fibre orientation distributions, and retention). Secondly, because of this continua approximations, the models are forced to use empirical laws, such as Darcy’s law [91], instead of first principles of physics. As a result, the models require a number of calibrations, losing their predictive power.

4.4. Results and discussion

Simulations were conducted to investigate the relation between the jet-to-wire speed difference and the paper structure, and the process itself. The effects on the fibre orientation anisotropy, the three-dimensionality of the sheet, and the basis weight variations were investigated. First, however, observations of the forming process itself are described.

4.4.1. Features of simulated dewatering

As the jet impinges on the wires, a pressure starts to build between them, which pushes fluid and rejects through them. The pressure gradient in the flow direction decelerates the suspension, predominantly in a region close to the impingement point. The sudden pressure drop at the point where the wires leaves the roll accelerates the suspension in the machine direction. Pressure pulses induced by the blades intermittently push fluid through the wires, but also cause a lateral pulsating flow of the suspension. The evolution of the fibre network structure is illustrated in Fig. 19, for the jet-to-wire speed difference $\Delta u_{jw} = 25$ m/min, and a jet thickness of 6.2 mm.

The development of the through-thickness concentration along the length of the forming section is illustrated in Fig. 20, for two different target basis weights (40 g/m² and 80 g/m²), but with the same suspension concentration. We observe that, for the lesser basis weight, there is no appreciable fibre concentration gradient during dewatering, except in the very final phase of drainage. For the greater basis weight, regions of slightly higher concentration develop near the wires during drainage on
the roll (see Fig. 20 at $t = 50$ ms). As the wires leave the roll, the sudden drop to atmospheric pressure induces an acceleration of the mix. The resulting high shear rate in the mix almost instantly flattens the concentration profile. Thus, if “filter cakes” develop on the roll, they may be destroyed when the wires leave the roll.

In Paper III, the drainage rate on the roll was estimated using the power law model of Ingemarsson [83], while the drainage rate over the blades was assumed to be proportional to the blade pressure. An alternative and more accurate method is to adjust the drainage rate in an iterative procedure, until the pressure profile of the drainage zone matches that of experiments. Figure 21 illustrated the pressure profile in the drainage zone, as computed directly in simulations, after the iteration procedure. In the plot, the pressure $p_h$ caused by hydrodynamic drag on the wires is plotted together with the structural pressure $p_s$, which was computed from the contact forces exerted by the fibres on the wires. Their sum $p_h + p_s$ represents the total dewatering pressure. The fluctuations of the hydrodynamic pressure over the roll may be due to the vortex-shedding of the wires, which was observed in the velocity field of these simulations, as well as in simulations performed by Huang et al. [92]. Also note that the simulation of Fig. 21 takes into account the sudden pressure drop at the point when the wires leave the roll; a phenomenon that was not taken into account in Paper III.

In the pressure data of Fig. 21, the structural pressure is negligible in comparison to the hydrodynamic drag on the wires, except at the final blade. When the hydrodynamic pressure of a 0.7% consistency suspension is compared with the hydrodynamic pressure for a simulation without fibres, but with the same imposed wire motion, it is seen that the presence of fibres significantly increases the hydrodynamic drag on the wires (see Fig. 21). The result indicates that additional hydrodynamic resistance develops from a synergy effect of the fibres and the wires.

The traditional view of dewatering in the paper machine is that filter cakes of fibrous material, so-called fibre mats, develop on the wires during forming. The
fibres are regions of distinctively higher solids concentration than the bulk of the suspension. Furthermore, these fibre mats are dense enough to support loads, so that the hydrodynamic pressure exerted on them by the fluid is transmitted to the wires through contact forces.

Inspired by this conceptual view, it has been suggested or assumed that the dewatering pressure in forming is governed by Darcy’s law [91], which was originally developed for predicting the pressure drop of water flowing through soil. In one dimension Darcy’s law takes the form

$$\Delta p = R_h \cdot Q,$$  \hspace{1cm} (4.1)

where $\Delta p$ is the pressure drop, $R_h$ is the hydrodynamic resistance, and $Q$ is the volumetric flux. A consequence of Eq. (4.1) is that the hydrodynamic resistance is additive; the total hydrodynamic resistance of many layers of soil is equal to the sum of the hydrodynamic resistance of each layer. When Darcy’s law is applied to dewatering in the paper machine, it is assumed a priori that fibre mats accumulate on the surfaces of the wires. Then, using the additivity of the hydrodynamic resistance, the dewatering pressure can be obtained as the sum of the pressure drop over the filter cake and the wire. However, two of the basic premises of Darcy’s law are that (i) the solid phase is stationary, and (ii) the typical sizes of the particles and capillaries are much smaller than the flow geometry considered, to allow for homogenisation. In the case of dewatering, condition (i) is violated because the concentration gradient is not stationary. Moreover, condition (ii) is violated because the fibre length and the floc size is of the same order as the thickness of the presumed filter cake. Consequently, Darcy’s law is questioned in the relevance to the process of dewatering in the paper machine. In fact, Bando and co-workers reported that for the experimentally determined $R_h$, not only volumetric flux, but also other process variables [93]. The simulations demonstrate explicitly that the additivity of hydrodynamic resistance does not hold; it was shown in Fig. 21 that the hydrodynamic

![Figure 21. Hydrodynamic pressure $p_h$ (thin, solid line), and structural pressure $p_s$ (thick, solid line) exerted on the wires at a consistency of 0.7 %, both plotted as functions of time. In addition, the hydrodynamic pressure for pure water (dashed line), with the same imposed motion of the wires, has been plotted for comparison.]
drag exerted on the wires increased by the mere presence of fibres. This is a synergy effect which cannot be explained in the context of Darcy’s law. There is no evidence that any significant structural pressure develops in the fibre phase, except at very high concentrations at the very final stage of dewatering.

In view of these results, it is reasonable that quantification of the dewatering resistance of pulps with the Schopper-Riegler number (SR) [94] or the Canadian standard freeness (CSF) [95] may not reflect the dewatering resistance in the machine, because the mechanisms of dewatering in small geometries are essentially different than the mechanisms at larger length scales. The simulation results suggest that the resistance to dewatering is not a property of the pulp, but rather a property of the system: the wires, the fibrous material, the fluid, the geometry, and the flow.

In contrast to the traditional view of dewatering in the paper machine, the present simulations also indicate that no well-defined dense region can be identified near the wires. An analogous result was recently presented by Sand et al. [96] for the development of the coating layer structure: In their simulations, the concentration profiles of coating pigments were almost constant with small variations during the development of the structure.

To the author’s knowledge, there are no direct measurements of the through-thickness concentration variations under realistic forming conditions. Neither are there any theoretical investigations available that makes clear the mechanisms behind the dewatering pressure development. This may open a new interesting challenge in the experimental forming research.

4.4.2. Fibre network structure

The jet-to-wire speed ratio was varied in the interval $\Delta u_{jw} \in [-50, 75]$ m/min by varying the jet speed, keeping the wire speed constant at 500 m/min. The fibre orientation anisotropy, the layeredness of the fibre network, the basis weight variations (formation), and the first pass fibre retention was calculated for the model sheets.

The anisotropy was quantified by least-squares fitting a general ellipse to a polar plot of the in-plane orientation distribution of fibres. The anisotropy was then taken as $1 - e$, where $e$ is the ratio between the minor and major axis of the ellipse. The anisotropy has been plotted as a function of the jet-to-wire speed difference in Fig. 22. The relation has a minimum at $\Delta u_{jw}^0 \approx 30$ m/min, which is known as the zero-point. The anisotropy increases in both the positive and negative direction from the zero-point, until it reaches plateaus. It was found that those plateaus roughly coincide with the maximum anisotropy that can be obtained if all fibres are aligned, but retain their equilibrium shape, which is not straight. Henceforth, the results will be presented with respect to the effective mix-to-wire speed $\Delta u_{mw} = \Delta u_{jw} - \Delta u_{jw}^0$, as suggested by Nordström and Norman [97] for the purpose of making comparisons between different experiments easier.

The virtual paper structure was delaminated numerically, and the fibre orientation anisotropy of each layer was computed to obtain the through-thickness anisotropy profile. It was investigated how this profile changes with mix-to-wire speed ratio. Figure 23 shows plots of the through-thickness anisotropy profile at different mix-to-wire speed differences. At the zero-point, the anisotropy profile is fairly constant through the thickness, with slight maxima near the surfaces of the sheet. At moderate positive or negative mix-to-wire speed differences, the anisotropy in-
Figure 22. The mean fibre orientation anisotropy $1 - e$ as a function of jet-to-wire speed difference $\Delta u_{jw}$. The minimum anisotropy is at the zero-point $\Delta u_{jw}^0$. The dashed line indicates the anisotropy that would be produced if all fibres were aligned in the same direction, but retained their equilibrium shape.

Figure 23. The through-thickness anisotropy profile for different mix-to-wire speed differences $\Delta u_{mw}$, where $\Delta u_{mw} = 0$ m/min Corresponds to the zero-point.

creases primarily near the surfaces. Keep in mind that drainage was symmetric in the simulations, leading to essentially symmetric anisotropy profiles, with small variations due to the element of randomness in the initial conditions. In a real sheet, two-sidedness would be expected due to asymmetric drainage.

The out-of-plane fibre orientation distribution of the sheet is difficult to access experimentally. (It is possible to obtain such detailed information using, for instance, automated microtoming [98].) Numerical paper descriptions provide complete access to every detail of the structure, and it is easy to quantify the three-dimensionality of the structure. In Paper III, the standard deviation $\sigma(\zeta)$ of the angle $\zeta$ between a fibre segment and the plane of the paper was used as a measure of the three-dimensionality. With this definition, $\sigma(\zeta) = \sqrt{\pi^2/4 - 2}$ rad corresponds to an isotropic orientation distribution, while $\sigma(\zeta) = 0$ rad corresponds to a perfectly layered structure. The simulations showed that the three-dimensionality is coupled to the anisotropy of the sheet: The three-dimensionality assumed its maximum at the zero-point. When the mix-to-wire speed difference was either increased or decreased, with respect to the zero-point, the three-dimensionality decreased and ultimately approached the level that would be expected if all fibres were aligned in the plane of the sheet, but retained their non-straight equilibrium shape. Although the
three-dimensionality measure is significantly reduced during the paper web consolidation downstream of the forming section, the feltedness/layeredness of the structure will be retained due to the geometrical constraints of the network.

*Formation* is a papermakers’ term for the degree of uniformity of the in-plane distribution of fibres. Different types of formation measures are described in the literature [99] to quantify the in-plane mass density distribution, which is also easily computed from virtual fibre networks. In Fig. 24, the formation can be judged by the eye from mass density plots of virtual papers produced at different effective mix-to-wire speed differences $\Delta u_{\text{mw}} = \Delta u_{\text{w}} - \Delta u_{\text{jw}}^0$. Formation can be quantified using the formation number $F(\lambda_1, \lambda_2)$, which is based on the variance spectrum of the mass density distribution in the wavelength interval $\lambda \in [\lambda_1, \lambda_2]$ (consult Paper III for the exact definition). The computed formation numbers of the simulated sheets have been plotted as a function of the mix-to-wire speed difference in Fig. 25(a). Nordström and Norman [100] used the STFI-method to measure formation, and obtained the formation number $F^*(0.3 \text{ mm}, 3 \text{ mm})$ experimentally (see Fig. 25(b)). Here, $F^*$ denotes a formation number with a slightly different definition suitable for analog measurements [99]. Therefore, $F(\lambda_1, \lambda_2)$ and $F^*(0.3 \text{ mm}, 3 \text{ mm})$ should not be compared quantitatively. Qualitatively, however, the formation of virtual papers and real papers vary with the mix-to-wire speed difference in the same way as real papers: with a minimum (optimum) formation number close to the zero-point. The scatter seen in Fig. 25(a) is due to the small size of the sample and the concentration variation of the jet.

A fraction of fibrous material, the rejects, passes through the wires. These fibres were eliminated from the simulations to somewhat reduce execution time. Therefore, they are not visible in Fig. 19. The first pass retention of fibres was taken as the mass of solids in the final sheet divided by mass of solids in the jet at the head-box nozzle tip. The relation between the effective mix-to-wire speed difference and the retention obtained from simulations has been plotted in Fig. 26. There is a relatively strong effect of the flow on the retention. The optimum retention, 95.5%, is found at approximately 4% rush. This behaviour may be specific to the particular set of process parameters, as reported in the literature [101]. It may also depend on the forming fabric geometry, in which case the simulations tool would be useful in forming fabric design.
Figure 24. Area density of simulated sheets obtained at different mix-to-wire speed differences $\Delta u_{mw}$. Bright areas correspond to high density. The bottom left sheet produced the minimum anisotropy in the investigated range of jet-to-wire speeds. The images heights are 5 mm in reality.
Figure 25. (a) Plot of the formation number $F$ of the simulated papers, in the wavelength interval $\lambda \in [0.3, 3.0]$ mm, as a function of effective mix-to-wire speed difference $\Delta u_{mw}$. The solid line has been drawn manually for guidance. (b) STFI formation $F^*$ in the wavelength interval $\lambda \in [0.3, 3.0]$ mm measured from real papers, manufactured in a roll-blade gap former (see Ref. [100]).

Figure 26. The simulated first pass fibre retention plotted as a function of effective mix-to-wire speed difference $\Delta u_{mw}$. Note that fines are not accounted for in the simulations.
5. CONCLUSIONS

A model has been proposed for simulating the dynamics of fibre suspensions. The validity range of the model includes the operating conditions of modern paper machines. The model was employed in simulations of roll–blade gap forming, to produce realistic fibre network structures, which were subsequently analysed.

In the proposed fibre suspension model, fibres are represented by chains of fibre segments, whose motion is governed by Newton’s second law. The motion of the fluid medium was calculated using the three-dimensional Navier–Stokes equations, and the mutual interactions between the fibres and the fluid were accounted for by enforcing momentum conservation between the solids and the fluid phases.

The most important additions in the new model, as compared to the previous model developed by Schmid et al. [40], are the inclusions of:

- A dynamic fluid flow, instead of a prescribed, stationary flow.
- The two-way coupling between the solids and the fluid phases.
- Particle inertia, i.e. finite Stokes numbers.
- Fibre–fluid interactions in the non-creeping flow regime, i.e. finite Reynolds numbers.

By including these features, the validity range of the model has been extended to flow conditions found in the forming section of modern paper machines. Furthermore, two major contributions were made to the computational efficiency of the fibre suspension model:

- The stability of the fibre model was analysed, and it was shown how to stabilise the equations of motion through artificial damping, while controlling the introduced error.
- The proposed fibre–fluid interaction model made possible a spatial discretisation of the Navier–Stokes equations, with a cell size in the order of the fibre segment length. The cost in execution time and memory usage was greatly reduced, as compared to Direct Numerical Simulations (DNS), for which the surface of the fibre must be resolved.

These two developments led to an improvement of the computational efficiency by orders of magnitude for the equations of motions of fibres, and the equations of motion of the fluid flow, respectively, while maintaining an accuracy of the predictions comparable to that of direct numerical simulations.

It was shown in Paper I, that the motion of isolated, rigid fibres in creeping shear flow is very well predicted by the proposed model. Without any parameter fitting procedure, the error in the predicted orbit period of isolated fibres in shear flow was less than 3.4%. In a second experiment, fair agreement was found between the simulated amount of bending for flexible fibres in springy and snakelike motion, and experimental data from the literature [72]. Consequently, the model quantitatively predicts the motion of isolated fibres in the rigid, springy and snakelike regimes of motion.
We proceed to summarise some of the conclusions that could be drawn from the simulations of suspensions of straight, rigid fibres subjected to shear flow, in Papers II and IV. The steady state orientation distribution was in good agreement with experimental data from the literature. Simulations were also able to predict a small but detectable asymmetry in the \( \phi \)-distribution throughout the semidilute regime, in turn indicating that mechanical fibre–fibre contact interactions are present. Indeed, there are theoretical grounds for suggesting that mechanical contacts are responsible for the development of finite first/second normal stress differences in the steady state (see Paper IV). It was indicated by theoretical arguments that it is the frequency of fibre–fibre contact events that control the development of the first/second normal stress differences. Then, this theory was corroborated through numerical experiments, and semi-empirical laws were established for the first/second normal stress difference and the apparent viscosity of well-dispersed suspensions.

After its validation in Papers I and III, the fibre suspension model was used to simulate forming of the fibre network with a roll–blade gap former. The jet-to-wire speed difference was varied and the simulated network structures were evaluated by computing the fibre orientation anisotropy, the layeredness of the network structure, and the basis weight variations. For each of these statistical measures, it was shown that their dependence on the jet-to-wire speed difference was in agreement with experimental results found in the literature.

Some questions were raised regarding the fundamental mechanisms governing the forming and dewatering. The traditional view on the development of the concentration gradient of fibrous material, and the dewatering pressure, is that they are essentially governed by the laws of macroscopic cake filtration. Simulations suggest that this may be an oversimplification. When the finite size of fibres and fibre flocs is taken into account, it becomes difficult to fit the microscopic behaviour of the fibre suspension into the continuum framework of cake filtration.

With this new model for simulation of the forming of the fibre network, it is straightforward to carry out parametric studies of any process parameter, including fibre morphology, configuration of drainage elements, etc., to investigate their effects on the fibre network structure. The simulated structures, in turn, can be used as input for further numerical studies of, for instance, the mechanical properties of the network [102].
### NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\dot{\gamma}$</td>
<td>Rate of strain tensor of the fluid, $\dot{\gamma} = \frac{1}{2} (\nabla v + (\nabla v)^T)$.</td>
</tr>
<tr>
<td>$\Delta p$</td>
<td>Pressure difference.</td>
</tr>
<tr>
<td>$\Delta t$</td>
<td>Time step of the simulations.</td>
</tr>
<tr>
<td>$\Delta u_{jw}$</td>
<td>Jet-to-wire speed difference.</td>
</tr>
<tr>
<td>$\Delta u_{jw}^0$</td>
<td>The zero-point, that is the jet-to-wire speed difference giving minimum anisotropy.</td>
</tr>
<tr>
<td>$\Delta u_{mw}$</td>
<td>Mix-to-wire speed difference $\Delta u_{mw} = \Delta u_{jw} - \Delta u_{jw}^0$.</td>
</tr>
<tr>
<td>$\Delta x$</td>
<td>Cell size of spatial discretisation of the fluid flow solver.</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Unit tensor.</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>Angle between fibre segment and the plane of the paper.</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Viscosity of the fluid medium.</td>
</tr>
<tr>
<td>$\eta_{\text{fib}}$</td>
<td>Fibre viscosity.</td>
</tr>
<tr>
<td>$\eta_{\text{sp}}$</td>
<td>Specific viscosity of the suspension.</td>
</tr>
<tr>
<td>$\eta_{\text{sp}}$</td>
<td>Steady state specific viscosity.</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Polar angle of the orientation vector, with the $x_3$-axis as the polar axis.</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Wavelength.</td>
</tr>
<tr>
<td>$\lambda_1$</td>
<td>Lower limit of wavelength interval.</td>
</tr>
<tr>
<td>$\lambda_2$</td>
<td>Upper limit of wavelength interval.</td>
</tr>
<tr>
<td>$\mu_f$</td>
<td>Coefficient of inter-fibre friction.</td>
</tr>
<tr>
<td>$\nu_t$</td>
<td>Kinematic eddy viscosity of a turbulent fluid.</td>
</tr>
<tr>
<td>$\xi$</td>
<td>Vorticity of the fluid medium.</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Density of the fluid medium.</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Stress tensor.</td>
</tr>
<tr>
<td>$\sigma(\zeta)$</td>
<td>Standard deviation of $\zeta$. Measure of the three-dimensionality of the fibre network.</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Azimuthal angle of the orientation vector, with the $x_3$-axis as the polar axis.</td>
</tr>
<tr>
<td>$\Psi$</td>
<td>Vector potential of the fluid velocity field.</td>
</tr>
<tr>
<td>$\Omega$</td>
<td>Box-shaped domain of computations.</td>
</tr>
<tr>
<td>$\Omega$</td>
<td>Angular velocity of the fluid, $\Omega = \frac{1}{2} \nabla \times v$.</td>
</tr>
<tr>
<td>$\omega_i$</td>
<td>Fibre coarseness. Dry fibre mass per unit length.</td>
</tr>
<tr>
<td>$\omega_i$</td>
<td>Angular velocity of fibre segment $i$.</td>
</tr>
<tr>
<td>$A_{i}^h$</td>
<td>Hydrodynamic resistance tensor of fibre segment $i$ for translational flow.</td>
</tr>
<tr>
<td>$C$</td>
<td>Orbit constant of Jeffery orbits.</td>
</tr>
<tr>
<td>$C_b$</td>
<td>Alternative orbit constant, $C_b = C/(1 + C)$.</td>
</tr>
<tr>
<td>$C_D$</td>
<td>Hydrodynamic drag coefficient.</td>
</tr>
<tr>
<td>$C_i^h$</td>
<td>Hydrodynamic resistance tensor of fibre segment $i$ for rotational flow.</td>
</tr>
<tr>
<td>$C_K$</td>
<td>Kolmogorov constant.</td>
</tr>
<tr>
<td>$c$</td>
<td>Volume concentration of fibres.</td>
</tr>
<tr>
<td>$c_m$</td>
<td>Solids concentration by mass.</td>
</tr>
<tr>
<td>$D$</td>
<td>Fibre diameter.</td>
</tr>
<tr>
<td>$d$</td>
<td>Eddy length scale.</td>
</tr>
<tr>
<td>Symbol</td>
<td>Quantity</td>
</tr>
<tr>
<td>--------</td>
<td>----------</td>
</tr>
<tr>
<td>$E_Y$</td>
<td>Young's modulus of fibre wall.</td>
</tr>
<tr>
<td>$e$</td>
<td>Ratio between minor and major axis of an ellipse fitted to the in-plane fibre orientation distribution.</td>
</tr>
<tr>
<td>$F$</td>
<td>Formation number (see Paper III).</td>
</tr>
<tr>
<td>$F^*$</td>
<td>Formation number obtained from analog measurements.</td>
</tr>
<tr>
<td>$F_2$</td>
<td>Second order velocity structure function.</td>
</tr>
<tr>
<td>$F^b$</td>
<td>Body force density acting on the fluid medium.</td>
</tr>
<tr>
<td>$F_{ih}$</td>
<td>Hydrodynamic force exerted on fibre segment $i$ by the fluid medium.</td>
</tr>
<tr>
<td>$F^p$</td>
<td>Force density representing the particle pressure exerted on the fluid.</td>
</tr>
<tr>
<td>$F_{iw}$</td>
<td>Body force exerted on fibre segment $i$.</td>
</tr>
<tr>
<td>$f_{ij}$</td>
<td>Interaction force exerted on fibre segment $i$ by fibre segment $j$ or a boundary.</td>
</tr>
<tr>
<td>$G$</td>
<td>Shear rate.</td>
</tr>
<tr>
<td>$g_0$</td>
<td>Dimensionless empirical constant.</td>
</tr>
<tr>
<td>$g_1$</td>
<td>Dimensionless empirical constant.</td>
</tr>
<tr>
<td>$H_{ih}$</td>
<td>Hydrodynamic resistance tensor of fibre segment $i$ for strain.</td>
</tr>
<tr>
<td>$h_1$</td>
<td>The $x_3$ position of the bottom forming fabric.</td>
</tr>
<tr>
<td>$h_2$</td>
<td>The $x_3$ position of the top forming fabric.</td>
</tr>
<tr>
<td>$I_i$</td>
<td>Tensor of inertia of fibre segment $i$.</td>
</tr>
<tr>
<td>$K$</td>
<td>Dimensionless empirical constant.</td>
</tr>
<tr>
<td>$L$</td>
<td>Fibre length.</td>
</tr>
<tr>
<td>$l_i$</td>
<td>Length of fibre segment $i$.</td>
</tr>
<tr>
<td>$m_i$</td>
<td>Mass of fibre segment $i$.</td>
</tr>
<tr>
<td>$N$</td>
<td>Number of fibre segment in a discretised fibre.</td>
</tr>
<tr>
<td>$N_1$</td>
<td>Dimensionless first normal stress difference.</td>
</tr>
<tr>
<td>$N_{1e}$</td>
<td>Steady state dimensionless first normal stress difference.</td>
</tr>
<tr>
<td>$N_2$</td>
<td>Dimensionless second normal stress difference.</td>
</tr>
<tr>
<td>$N_{2e}$</td>
<td>Steady state dimensionless second normal stress difference.</td>
</tr>
<tr>
<td>$N_c$</td>
<td>Crowding factor (see Ref. [55]).</td>
</tr>
<tr>
<td>$n$</td>
<td>Number of fibres per unit volume.</td>
</tr>
<tr>
<td>$P_L$</td>
<td>Intensity of flocculation.</td>
</tr>
<tr>
<td>$P_{Le}$</td>
<td>Steady state intensity of flocculation.</td>
</tr>
<tr>
<td>$\hat{p}$</td>
<td>Orientation vector; unit vector pointing along the length of a straight fibre.</td>
</tr>
<tr>
<td>$p$</td>
<td>Pressure.</td>
</tr>
<tr>
<td>$p(\phi)$</td>
<td>Azimuthal angle distribution.</td>
</tr>
<tr>
<td>$p(C_b)$</td>
<td>Orbit constant distribution.</td>
</tr>
<tr>
<td>$p^+(\phi)$</td>
<td>Alternative azimuthal angle distribution.</td>
</tr>
<tr>
<td>$p_h$</td>
<td>Pressure caused by hydrodynamic drag on the wire yarns.</td>
</tr>
<tr>
<td>$p_m$</td>
<td>Pressure between the wires during drainage.</td>
</tr>
<tr>
<td>$p_s$</td>
<td>Structural pressure caused by fibre–wire contacts.</td>
</tr>
<tr>
<td>$Q$</td>
<td>Volumetric flux.</td>
</tr>
<tr>
<td>$Re$</td>
<td>Reynolds number.</td>
</tr>
<tr>
<td>$Re_p$</td>
<td>Particle Reynolds number for fibre–fluid cross-flow.</td>
</tr>
<tr>
<td>Symbol</td>
<td>Quantity</td>
</tr>
<tr>
<td>--------</td>
<td>----------</td>
</tr>
<tr>
<td>$R_h$</td>
<td>Hydrodynamic resistance.</td>
</tr>
<tr>
<td>$r_e$</td>
<td>Equivalent aspect ratio.</td>
</tr>
<tr>
<td>$r_i$</td>
<td>Position vector pointing to the centroid of fibre segment $i$.</td>
</tr>
<tr>
<td>$r_p$</td>
<td>Fibre aspect ratio, $r_p = L/D$.</td>
</tr>
<tr>
<td>$St$</td>
<td>Stokes number.</td>
</tr>
<tr>
<td>$s^2_L(\ldots)$</td>
<td>Variance of local quantity over many volumes $L^3$.</td>
</tr>
<tr>
<td>$T^h_i$</td>
<td>Hydrodynamic torque exerted on fibre segment $i$ by the fluid medium.</td>
</tr>
<tr>
<td>$t$</td>
<td>Time.</td>
</tr>
<tr>
<td>$t_{ij}$</td>
<td>Interaction torque caused by the interaction force $f_{ij}$.</td>
</tr>
<tr>
<td>$u_f$</td>
<td>Fibre–fluid cross-flow magnitude.</td>
</tr>
<tr>
<td>$\Delta u_{mw}$</td>
<td>Mix-to-wire speed difference.</td>
</tr>
<tr>
<td>$u_{tip}$</td>
<td>Headbox flow velocity magnitude at vane tips.</td>
</tr>
<tr>
<td>$v$</td>
<td>Velocity of the fluid medium.</td>
</tr>
<tr>
<td>$v_0$</td>
<td>Initial fluid velocity field.</td>
</tr>
<tr>
<td>$x_i$</td>
<td>Coordinate in a right-hand coordinate system, $i = 1, 2, 3$.</td>
</tr>
<tr>
<td>$X_i$</td>
<td>Connectivity force exerted on segment $i - 1$ by segment $i$.</td>
</tr>
<tr>
<td>$Y_i$</td>
<td>Bending moment and torsional torque exerted on segment $i - 1$ by segment $i$.</td>
</tr>
<tr>
<td>$\hat{z}_i$</td>
<td>Unit vector pointing in the length direction of fibre segment $i$.</td>
</tr>
</tbody>
</table>
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REFERENCES


